Handout for Lecture 10

Normal Distribution and Z-Score

ECON 340: Economic Research Methods — Instructor: Div Bhagia

If $X \sim N(\mu, \sigma^2)$, then the standardized random variable,

$$Z = \frac{X - \mu}{\sigma} \sim N(0, 1)$$

Given $X \sim N(\mu, \sigma^2)$, to find $Pr(x_0 < X < x_1)$:

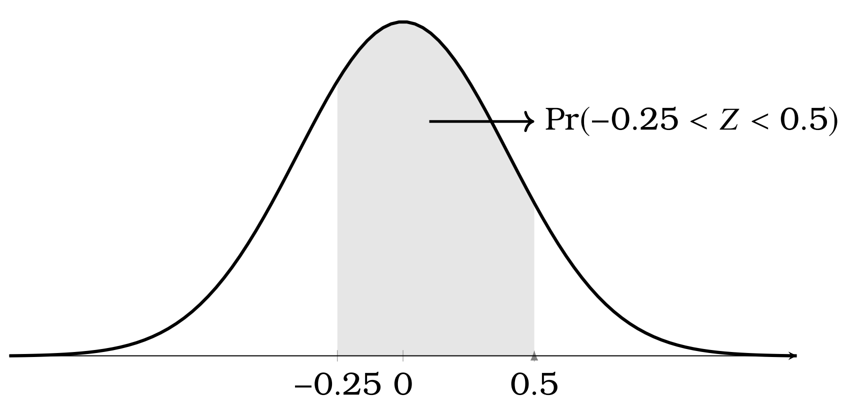
- Find $z_0 = (x_0 - \mu)/\sigma$ and $z_1 = (x_1 - \mu)/\sigma$
- Use standard normal table to find $Pr(z_0 < Z < z_1)$

Exercises: Refer to the standard normal table to answer the following.

1. Given $X \sim N(3, 16)$, find $Pr(2 < X < 5)$.

$$\Pr(2 < X < 5) = \Pr\left(\frac{2-3}{4} < Z < \frac{5-3}{4}\right) = \Pr(-0.25 < Z < 0.5)$$

From the standard normal table, $\Pr(Z < -0.25) = 0.4013$ and $\Pr(Z < -0.5) = \Pr(Z > 0.5) = 0.3085$. So $\Pr(-0.25 < Z < 0.5) = 1 - 0.4013 - 0.3085 = 0.2902$.



2. Given $X \sim N(15, 100)$, find $Pr(X > -3)$.

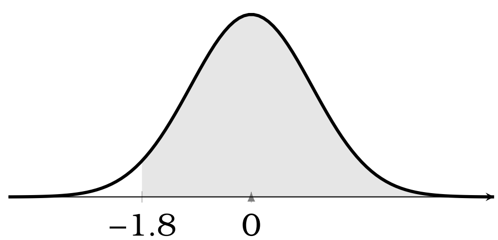
$$\Pr(X > -3) = \Pr\left(\frac{X-15}{10} > \frac{-3-15}{10}\right) = \Pr(Z > -1.8)$$

Note that,

$$\Pr(Z > -1.8) = 1 - \Pr(Z < -1.8) = 1 - 0.0359 = 0.9641$$

Here, I got $\Pr(Z < -1.8)$ from the standard normal table. Alternatively,

$$\Pr(Z > -1.8) = \Pr(Z < 1.8) = 0.9641$$



Given $X \sim N(\mu, \sigma^2)$ and $Pr(X < x) = p$, to find x :

- Use standard normal table to find z where $Pr(Z < z) = p$
- Find $x = \mu + z \cdot \sigma$

Follows analogously for when we are given $Pr(X > x) = p$.

Exercises: Refer to the standard normal table to answer the following.

1. Given $Pr(Z > z) = 0.95$. Find z.

   From the standard normal table, $\Pr(Z < 1.645) = 0.95$. Since the normal distribution is symmetric, $\Pr(Z > -1.645) = 0.95$. So, $z = -1.645$

2. Given $X \sim N(3, 16)$ and $Pr(X < x) = 0.95$. Find x.

   From the standard normal table, $\Pr(Z < 1.645) = 0.95$. Note that,

   $$Z = \frac{X - \mu}{\sigma} \rightarrow X = \mu + Z.\sigma$$

   So here, $x = 3 + 1.645 \times 4 = 9.58$.

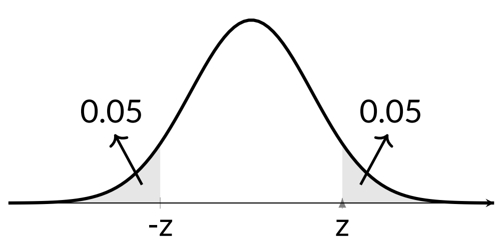
3. Given $Pr(|Z| > z) = 0.10$. Find z.

   Note: Since the normal distribution is symmetric $Pr(Z > z) = Pr(Z < -z)$, so we have that: $Pr(|Z| > z) = 2Pr(Z > z) = 2Pr(Z < -z)$.

   First note that,

   $$Pr(|Z| > z) = Pr(Z > z) + Pr(Z < -z) = 2Pr(Z < -z)$$

   Since, $Pr(|Z| > z) = 0.1$, we are trying to find a z such that $Pr(Z < -z) = 0.05$.



   From the standard normal table, $z = 1.645$.