Handout for Lecture 11
Independence and Correlation

ECON 340: Economic Research Methods — Instructor: Div Bhagia

Consider two random variables X and Y.

- The *joint probability* $f(x, y) = Pr(X = x, Y = y)$ represents the likelihood that X equals x and Y equals y.
- The marginal probability of $Y = y$, denoted $f(y)$, is obtained by summing the joint probability $f(x, y) = Pr(X = x, Y = y)$ over all possible values of x.
- The *conditional probability* $f(y|x) = Pr(Y = y|X = x)$ represents the likelihood that Y is equal to y, given that X is equal to x.

$$f(y|x) = Pr(Y = y|X = x) = \frac{Pr(X = x, Y = y)}{Pr(X = x)} = \frac{f(x, y)}{f(x)}$$

- The *conditional expectation* $E(Y|x)$ is the expected value of Y given that $X = x$.

$$E(Y|X = x) = \sum_y yPr(Y = y|X = x) = \sum_y yf(y|x)$$

Uncorrelated vs Independent

- Covariance and correlation:

$$\sigma_{XY} = Cov(X, Y) = E[(X - \mu_X)(Y - \mu_Y)]$$

$$\rho_{XY} = corr(X, Y) = \frac{\sigma_{XY}}{\sigma_X \sigma_Y} \quad \text{where } -1 \leq \rho \leq 1$$

- Two random variables are *uncorrelated* if $corr(X, Y) = 0$.
- Two random variables are *independent* if $f(y|x) = f(y)$ for all x and y or equivalently $E(Y|X) = E(Y)$.
- If X and Y are independent, then they are also uncorrelated. (The converse is not necessarily true.)

1. The following table gives the joint-probability distribution of rain (X) *and* commute time in minutes (Y).

	Rain $(X = 1)$	No Rain $(X = 0)$	Total
60-min commute ($Y = 60$)	0.3	0.2	
30-min commute ($Y = 30$)	0.1	0.4	
Total			

   (a) Fill the marginal probabilities in the column and row labeled *Total*. For example, the first row in column *Total* should contain the marginal probability of having a 60-minute commute ($Pr(Y = 60)$).

   (b) Find the following conditional probabilities:

   - Probability of having a 60-min commute conditional on *raining*

     $Pr(Y = 60|X = 1) =$

   - Probability of having a 60-min commute conditional on *not raining*

     $Pr(Y = 60|X = 0) =$

   (c) Calculate $E(Y|X = 1)$ and $E(Y|X = 0)$.

(d) How does rain impact the expected commute time in this example?

2. You flipped a coin six times and got tails each time. The likelihood of getting a head in your seventh flip is

   - ☐ 1/2
   - ☐ More than 1/2
   - ☐ Less than 1/2

3. Note that for sums of two random variables X and Y:

$$E(aX + bY) = aE(X) + bE(Y)$$

$$Var(aX + bY) = a^2Var(X) + b^2Var(Y) + 2abCov(X, Y)$$

   In the above expressions, a and b are constants.

   You are contemplating investing in two stocks with the same average return and spread.

$$\mu_X = \mu_Y \qquad \sigma_X^2 = \sigma_Y^2$$

   Should you pick any one stock at random or invest equally in both?

   Hint: Let $W = 0.5X + 0.5Y$.