Midterm Exam: Help Sheet

ECON 441: Introduction to Mathematical Economics — Instructor: Div Bhagia

Linear Algebra

The **minor** of the element a_{ij}, denoted by $|M_{ij}|$ is obtained by deleting the ith row and jth column of the matrix and taking the determinant of the resulting matrix.

Whereas, **cofactor** $|C_{ij}|$ is defined as:

$$|C_{ij}| = (-1)^{i+j}|M_{ij}|$$

Determinant for an $n \times n$ matrix when expanding with respect to the *first row* is given by:

$$|A| = \sum_{j=1}^{n} a_{1j}|C_{1j}|$$

To find the **inverse** of matrix A take the transpose of its cofactor matrix $C = [|C_{ij}|]$ to find the adjoint of A and divide it by the determinant of A.

$$A^{-1} = \frac{1}{|A|} adj A$$

Adjoint of a $n \times n$ matrix

$$adj A = C' = \begin{bmatrix} |C_{11}| & |C_{21}| & \dots & |C_{n1}| \\ |C_{12}| & |C_{22}| & \dots & |C_{n2}| \\ \vdots & \vdots & \dots & \vdots \\ |C_{1n}| & |C_{2n}| & \dots & |C_{nn}| \end{bmatrix}$$

Cramer's rule: Given $Ax = b$, form matrix A_k by interchanging the k^{th} column of A by b, then

$$x_k^* = \frac{|A_k|}{|A|}$$

Comparative Statics

Limit Definition of the Derivative

$$\frac{dy}{dx} = f'(x_0) = \lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

Limit of a function

We say the limit of a function $f(x)$ exists at $x = a$ if both the right-side and left-side limits at a are equal.

Continuity of a Function

A function $y = f(x)$ is said to be continuous at a if the limit of $f(x)$ at a exists and is equal to the value of the function at a i.e., $\lim_{x \to a} f(x) = f(a)$.

Product Rule

$$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + f'(x)g(x)$$

Quotient Rule

$$\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$$

Inverse Function Rule

For $y = f(x)$ and $x = f^{-}1(y)$

$$\frac{dy}{dx} = \frac{1}{dx/dy}$$

Chain Rule

For functions $z = f(y)$ and $y = g(x)$,

$$\frac{dz}{dx} = \frac{dz}{dy} \cdot \frac{dy}{dx} = f'(y)g'(x)$$

Derivative of Exponential & Log function

$$\frac{d}{dx}e^x = e^x \qquad \frac{d}{dx}\ln x = \frac{1}{x}$$

Partial Derivative

If x_1 changes by Δx_1 but all other variables remain constant:

$$\frac{\Delta y}{\Delta x_1} = \frac{f(x_1 + \Delta x_1, x_2, \cdots, x_n) - f(x_1, x_2, \cdots, x_n)}{\Delta x_1}$$

The partial derivative of y with respect to x_i:

$$\frac{\partial y}{\partial x_i} = f_i = \lim_{\Delta x_i \to 0} \frac{\Delta y}{\Delta x_i}$$

Gradient Vector

$$\nabla f(x_1, x_2, \cdots, x_n) = [f_1, f_2, \cdots, f_n]'$$

Elasticity

$$\varepsilon = \frac{\text{Percentage change in } y}{\text{Percentage change in } x} = \frac{dy/y}{dx/x} = \frac{dy}{dx} \cdot \frac{x}{y}$$

Total differential

$$df = \frac{\partial f}{\partial x_1} dx_1 + \frac{\partial f}{\partial x_2} dx_2 + \cdots + \frac{\partial f}{\partial x_n} dx_n = \sum_{i=1}^{n} f_i dx_i$$

Total derivative

Total derivative with respect to x_1:

$$\frac{df}{dx_1} = \frac{\partial f}{\partial x_1} + \frac{\partial f}{\partial x_2}\frac{dx_2}{dx_1} + \cdots + \frac{\partial f}{\partial x_n}\frac{dx_n}{dx_1}$$

If x_i doesn't depend on x_1 then $dx_i/dx_1 = 0$. If f does not directly depend on x_1 then $\partial f/\partial x_1 = 0$.