

Homework 1 Problems

ECON 441: Introduction to Mathematical Economics

Instructor: Div Bhagia

Exercise 2.3

1. Write the following in set notation:

- (a) The set of all real numbers greater than 34.
- (b) The set of all real numbers greater than 8 but less than 65.

2. Given the sets $S_1 = \{2, 4, 6\}$, $S_2 = \{7, 2, 6\}$, $S_3 = \{4, 2, 6\}$, and $S_4 = \{2, 4\}$, which of the following statements are true?

- | | | |
|----------------------|----------------------------|-----------------------------|
| a) $S_1 = S_3$ | b) $S_1 = \mathbb{R}$ | c) $8 \in S_2$ |
| d) $3 \notin S_2$ | e) $4 \notin S_3$ | f) $S_4 \subset \mathbb{R}$ |
| g) $S_1 \supset S_4$ | h) $\emptyset \subset S_2$ | i) $S_3 \supset \{1, 2\}$ |

Note that \mathbb{R} denotes the set of real numbers.

Exercise 2.4

5. If the domain of the function $y = 5 + 3x$ is the set $\{x | 1 \leq x \leq 9\}$, find the range of the function and express it as a set.

7. In the theory of the firm, economists consider the total cost C to be a function of the output level Q : $C = f(Q)$.

- (a) According to the definition of a function, should each cost figure be associated with a unique output level?
- (b) Should each level of output determine a unique cost figure?

8. If an output level Q_1 can be produced at a cost of C_1 , then it must also be possible (by being less efficient) to produce Q_1 at a cost of $C_1 + \$1$, or $C_1 + \$2$, and so on. Thus it would seem that output Q does not uniquely determine total cost C . If so, to write $C = f(Q)$ would violate the definition of a function. How, in spite of this reasoning, would you justify the use of the function $C = f(Q)$?

Exercise 2.5

1. Graph the following functions and find their inverse functions.

(a) $y = 16 + 2x$

(b) $y = 8 - 2x$

(c) $y = 2x + 12$

Exercise 4.2

6. Expand the following summation expressions:

(a) $\sum_{i=2}^5 x_i$

(b) $\sum_{i=5}^8 a_i x_i$

(c) $\sum_{i=1}^4 b x_i$

(d) $\sum_{i=1}^n a_i x^{i-1}$

(e) $\sum_{i=0}^3 (x + i)^2$

8. Show that the following are true:

(a) $\left(\sum_{i=0}^n x_i \right) + x_{n+1} = \sum_{i=0}^{n+1} x_i$

(b) $\sum_{j=1}^n a b_j y_j = a \sum_{j=1}^n b_j y_j$

(c) $\sum_{j=1}^n (x_j + y_j) = \sum_{j=1}^n x_j + \sum_{j=1}^n y_j$

Exercise 5.1

1. In the following paired statements, let p be the first statement and q the second.

Which is true for each case: $p \Rightarrow q$, $p \Leftarrow q$, or $p \Leftrightarrow q$?

(a) It is a holiday; it is Thanksgiving Day.

(b) A geometric figure has four sides; it is a rectangle.

(c) Two ordered pairs (a, b) and (b, a) are equal; a is equal to b .

(d) A number is rational; it can be expressed as a ratio of two integers.

(e) A 4×4 matrix is nonsingular; the rank of the 4×4 matrix is 4. (*skip for now*)

(f) The gasoline tank in my car is empty; I cannot start my car.

(g) The letter is returned to the sender with the marking "addressee unknown"; the sender wrote the wrong address on the envelope.