Homework 2 Problems

ECON 441: Introduction to Mathematical Economics — Instructor: Div Bhagia

Exercise 4.2

1. Given $A = \begin{bmatrix} 7 & -1 \\ 6 & 9 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 4 \\ 3 & -2 \end{bmatrix}$, and $C = \begin{bmatrix} 8 & 3 \\ 6 & 1 \end{bmatrix}$, find:

   (a) $A + B$ (b) $C - A$

   (c) $3A$ (d) $4B + 2C$

2. Given $A = \begin{bmatrix} 2 & 8 \\ 3 & 0 \\ 5 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 0 \\ 3 & 8 \end{bmatrix}$, and $C = \begin{bmatrix} 7 & 2 \\ 6 & 3 \end{bmatrix}$:

   (a) Is AB defined? Calculate AB. Can you calculate BA? Why?

   (b) Is BC defined? Calculate BC. Is CB defined? If so, calculate CB. Is it true that $BC = CB$.

4. Find the product matrices in the following (in each case, append beneath every matrix a dimension indicator):

   (a) $\begin{bmatrix} 0 & 2 & 0 \\ 3 & 0 & 4 \\ 2 & 3 & 0 \end{bmatrix} \begin{bmatrix} 8 & 0 \\ 0 & 1 \\ 3 & 5 \end{bmatrix}$ (b) $\begin{bmatrix} 6 & 5 & -1 \\ 1 & 0 & 4 \end{bmatrix} \begin{bmatrix} 4 & -1 \\ 5 & 2 \\ 0 & 1 \end{bmatrix}$

   (c) $\begin{bmatrix} 3 & 5 & 0 \\ 4 & 2 & -7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ (d) $\begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} 7 & 0 \\ 0 & 2 \\ 1 & 4 \end{bmatrix}$

Exercise 4.4

5. (e) Find (i) $C = AB$, and (ii) $D = BA$, if

$$A = \begin{bmatrix} -2 \\ 4 \\ 7 \end{bmatrix} \quad B = \begin{bmatrix} 3 & 6 & -2 \end{bmatrix}$$

7. If the matrix A in Example 5 had all its four elements nonzero, would $x'Ax$ still give a weighted sum of squares? Would the associative law still apply?

Exercise 4.5

1. Given $A = \begin{bmatrix} -1 & 5 & 7 \\ 0 & -2 & 4 \end{bmatrix}$, $b = \begin{bmatrix} 9 \\ 6 \\ 0 \end{bmatrix}$, and $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$:

   Calculate: (a) AI (b) IA (c) Ix (d) $x'I$
   Indicate the dimension of the identity matrix used in each case.

4. Show that the diagonal matrix

$$\begin{bmatrix} a_{11} & 0 & \cdots & 0 \\ 0 & a_{22} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & a_{nn} \end{bmatrix}$$

   can be idempotent only if each diagonal element is either 1 or 0 . How many different numerical idempotent diagonal matrices of dimension $n \times n$ can be constructed altogether from such a matrix?

Exercise 4.6

2. Given $A = \begin{bmatrix} 0 & 4 \\ -1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -8 \\ 0 & 1 \end{bmatrix}$, and $C = \begin{bmatrix} 1 & 0 & 9 \\ 6 & 1 & 1 \end{bmatrix}$, verify that
   (a) $(A+B)' = A' + B'$
   (b) $(AC)' = C'A'$

6. Let $A = I - X\,(X'X)^{-1}\,X'$.
   (a) Must A be square? Must $(X'X)$ be square? Must X be square?
   (b) Show that matrix A is idempotent.