Homework 7 Solutions

ECON 441: Introduction to Mathematical Economics — Instructor: Div Bhagia

Exercise 8.5

1. For each $F(x, y) = 0$, find dy/dx for each of the following:

   (a) $y - 6x + 7 = 0$

$$\frac{dy}{dx} - 6 = 0 \rightarrow \frac{dy}{dx} = 6$$

   (b) $3y + 12x + 17 = 0$

$$3\frac{dy}{dx} + 12 = 0 \rightarrow \frac{dy}{dx} = \frac{-12}{3} = -4$$

   (c) $x^2 + 6x - 13 - y = 0$

$$2x + 6 - \frac{dy}{dx} = 0 \rightarrow \frac{dy}{dx} = 2x + 6$$

2. (d) $F(x, y) = 6x^3 - 3y = 0$. By implicit function theorem:

$$\frac{dy}{dx} = \frac{-F_x}{F_y}$$

$$F_x = 18x^2, \quad F_y = -3$$

$$\frac{dy}{dx} = \frac{-18x^2}{-3} = 6x^2$$

3. (a) $F(x, y, z) = x^2y^3 + z^2 + xyz = 0$. By implicit function theorem:

$$\frac{\partial y}{\partial x} = \frac{-F_x}{F_y} = \frac{-\left(2xy^3 + yz\right)}{3x^2y^2 + xz}$$

$$\frac{\partial y}{\partial z} = \frac{-F_z}{F_y} = \frac{-(2z + xy)}{3x^2y^2 + xz}$$

Exercise 14.2

1. Find the following:

(a) $\int 16x^{-3}\, dx = \frac{16x^{-2}}{-2} + c = -8x^{-2} + c \quad (x \neq 0)$

(c) $\int (x^5 - 3x)\, dx = \int x^5\, dx - 3\int x\, dx = \frac{x^6}{6} - \frac{3x^2}{2} + c$

(d) $\int 2e^{-2x}\, dx = 2\int e^{-2x}\, dx = 2\frac{e^{-2x}}{-2} + c = -e^{-2x} + c$

Exercise 14.3

1. Evaluate the following:

(a) $\int_1^3 \frac{1}{2}x^2 = \left[\frac{x^3}{6}\right]_1^3 = \frac{3^3 - 1^3}{6} = \frac{26}{6}$

(e) $\left[\frac{ax^3}{3} + \frac{bx^2}{2} + cx\right]_{-1}^{1} = \left(\frac{a}{3} + \frac{b}{2} + c\right) - \left(-\frac{a}{3} + \frac{b}{2} - c\right) = 2\left(\frac{a}{3} + c\right)$

2. Evaluate the following:

(a) $\int_1^2 e^{-2x}\, dx = \left[\frac{-e^{-2x}}{2}\right]_1^2 = -\frac{1}{2}\left(e^{-4} - e^{-2}\right) = \frac{1}{2}\left(e^{-2} - e^{-4}\right)$

(d) $[\ln x + \ln(1 + x)]_e^6 = \ln 6 + \ln 7 - \ln e - \ln(1 + e) = \ln 42 - 1 - \ln(1 + e)$

5. Verify that a constant c can be equivalently expressed as a definite integral:

(a) $\int_0^b \frac{c}{b}\, dx = \left[\frac{cx}{b}\right]_0^b = \frac{cb}{b} - \frac{c.0}{b} = c$

(b) $\int_0^c 1\, dt = [t]_0^c = c - 0 = c$