

DURATION DEPENDENCE AND HETEROGENEITY: LEARNING FROM EARLY NOTICE OF LAYOFF*

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Abstract

This paper presents a novel approach to distinguish the impact of duration-dependent forces and adverse selection on the exit rate from unemployment by leveraging variation in the length of layoff notices. I formulate a Mixed Hazard model in discrete time and specify the conditions under which variation in notice length enables the identification of structural duration dependence while allowing for arbitrary heterogeneity across workers. Utilizing data from the Displaced Worker Supplement (DWS), I employ the Generalized Method of Moments (GMM) to estimate the model. According to the estimates, the decline in the exit rate over the first 48 weeks of unemployment is largely due to the worsening composition of surviving jobseekers. Furthermore, I find that an individual's likelihood of exiting unemployment decreases initially, then increases until unemployment benefits run out, and remains steady thereafter. These findings are consistent with a standard search model where returns to search decline early in the spell.

Keywords: *Duration dependence, unemployment, exit rate, job-finding rate, unobserved heterogeneity, hazard models, advance notice*

JEL Codes: *C41, E24, J64, J65*

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I INTRODUCTION

A well-established empirical regularity is that the exit rate out of unemployment decreases over the spell of unemployment, except for a spike at the time of unemployment insurance (UI) exhaustion. The decline in the exit rate may represent negative duration dependence, meaning that the longer a worker remains unemployed, the less likely they are to exit unemployment. This would be true if employers discriminate against long-term unemployed workers or if workers lose valuable skills and connections over time, which would otherwise assist them in finding employment. However, workers with different unemployment durations, who may appear similar to researchers, may actually be quite different from each other. Factors such as employability, the urgency to find a job, or the ability to secure employment may vary across individuals. Such heterogeneity across workers would imply that the observed exit rate declines even in the absence of structural duration dependence. As more employable workers leave unemployment, the remaining pool of unemployed individuals increasingly consists of those who are less likely to exit unemployment.

Understanding how the likelihood of exiting unemployment evolves over the unemployment spell and the extent of heterogeneity across workers is crucial for the design of unemployment policies.¹ Furthermore, the magnitude and direction of structural duration dependence have implications for the incidence of long-term unemployment and the speed of recovery from economic downturns (Pissarides, 1992). Given its significance, a substantial body of literature has attempted to disentangle the sources of the decline in the exit rate from unemployment. However, it has proven to be challenging to do so using observational data.

In this paper, I develop and implement a novel approach to empirically disentangle

¹See Shimer and Werning (2006), Pavoni and Violante (2007), Pavoni (2009), and Kolsrud et al. (2018).

the contributions of structural duration dependence and unobserved heterogeneity in explaining the dynamics of the exit rate from unemployment. My approach relies on leveraging variation in the length of notice workers receive from their employers before being laid off. Using data from the Displaced Worker Supplement (DWS), I compare workers with a notice period of more than two months (referred to as long notice) to workers with a notice period of less than two months (referred to as short notice). To ensure comparability across the two groups, I use inverse probability weighting (IPW) to achieve balance on a comprehensive set of observable characteristics. The analysis reveals that during the initial 12 weeks, the exit rate out of unemployment is 7.4 percentage points higher for long-notice workers. This difference is primarily due to a larger proportion of long-notice workers transitioning directly to their next job without experiencing a period of unemployment. However, beyond the first 12 weeks, the exit rate for workers with the longer notice is actually lower.

I argue that the lower exit rate for long-notice workers at later durations is due to the composition of this group becoming relatively worse as a larger proportion of individuals exit early in the spell. This indicates the presence of heterogeneity among workers. In the presence of heterogeneity, such as differences in employability, those who are more employable exit unemployment earlier. As more workers exit early from the long-notice group, the surviving workers from this group will have a lower proportion of highly employable workers compared to the short-notice group. Conversely, if there is no heterogeneity, a larger proportion of workers exiting early from the long-notice group will not alter the composition of long versus short-notice workers at later durations. Consequently, there would be no discernible difference in the exit rates of the two groups.² Thus, the difference in the exit rates of short and long-notice workers is indicative of the extent of underlying heterogeneity. This is the fundamental idea behind my approach, which enables me to pin down the extent of heterogeneity and

²It is also possible that receiving a longer notice directly affects a worker's exit probability, even at later durations. I discuss this possibility below while addressing the robustness of my findings.

estimate structural duration dependence.

I operationalize this intuition by formulating a Mixed Hazard (MH) model (Lancaster, 1979) in discrete time with multiple notice lengths. Within this framework, I specify the probability of an individual exiting unemployment as a product of their unobservable type, a function of observable factors, and a structural hazard that varies with the duration of unemployment and notice length. I show that structural duration dependence, characterized by how the structural hazard varies with the duration of unemployment, can be identified when two conditions hold. The first condition, commonly referred to as unconfoundedness (Rosenbaum and Rubin, 1983), requires that the length of notice is independent of the worker's unobservable type when conditioned on observable characteristics. The second condition states that the length of the notice period does not impact the structural hazard at later durations of unemployment. In other words, while a longer notice period before layoff may affect the probability of exiting at the beginning of the unemployment spell, it does not influence the probability of exiting at later durations. Building on the identification result, I develop a method for estimating the model using the Generalized Method of Moments (GMM). The estimation method utilizes moments that are weighted to ensure that the distribution of observable characteristics is similar for different notice lengths.

Relative to the existing literature on the identification and estimation of the Mixed Hazard model, my approach goes further in several dimensions. Firstly, I do not impose any functional form restrictions on the distribution of heterogeneity. This is crucial because misspecification of unobserved heterogeneity can significantly impact estimates of structural duration dependence, as demonstrated by Heckman and Singer (1984). Secondly, identification in my model stems from variation in a variable—the length of notice—that is exogenous conditional on observables and is assumed to affect the structural hazard in a way that aligns with economic intuition.³ The introduction of uncon-

³Existing non-parametric identification results for the Mixed Hazard model rely on variation in an

foundedness in the Mixed Hazard framework is a novel innovation. Lastly, I provide a root- n consistent estimator for the parameters of my model. The framework I employ is analogous to a Mixed Hazard model with a time-varying exogenous variable. While [Brinch \(2007\)](#) provides a non-constructive proof for this model in continuous time, the key distinction here is that the exposition is in discrete time, which leads to a consistent estimator for the model's parameters using GMM. To the best of my knowledge, [Alvarez et al. \(2021\)](#) is the only other study that utilizes moment conditions from a discrete version of the Mixed Hazard model and constructs a GMM estimator. However, their identification result and estimator pertain to multiple spell data.⁴

I estimate the model using weighted moments from the DWS data. The estimates uncover substantial heterogeneity in individual exit probabilities. I find that about half of the 41% decline in the observed exit rate over the first five months is due to the changing composition of workers over the spell of unemployment. Moreover, I find that after the first five months, an individual worker's exit probability increases until the time of their unemployment benefit exhaustion and remains constant after. This is in contrast to the observed exit rate, which continues to decline even after benefit exhaustion. Recently, researchers have proposed behavioral modifications to standard search theory to explain this decline ([Boone and van Ours, 2012](#); [DellaVigna et al., 2017, 2021](#)). However, I offer an alternative explanation: as a substantial number of individuals exit unemployment right at the point of benefit exhaustion, the composition of the remaining unemployed workers becomes significantly worse. This compositional change contributes to the observed decline in the exit rate after benefit exhaustion. Finally, I calibrate a partial equilibrium search model with a non-stationary environment

exogenous variable that enters the structural hazard multiplicatively ([Elbers and Ridder, 1982](#); [Heckman and Singer, 1984](#)). The practical implementation of these results has been limited due to the challenge of locating a variable that meets this criterion, as well as the absence of a convenient estimator. Another approach to identification is using multiple spell data ([Honoré, 1993](#)). However, this approach assumes that the unobserved characteristics of the jobseeker remain constant across repeated spells.

⁴[van den Berg and van Ours \(1996\)](#) also set up a discrete-time MPH model. However, they do not derive the distribution of their estimator.

(Mortensen, 1986; Van Den Berg, 1990) and show that my findings can be rationalized in this framework with a decline in returns to search early in the spell.

Under the identifying assumptions specified for the Mixed Hazard model in my framework, the lower exit rate among long-notice workers after the initial 12 weeks is attributed to the presence of heterogeneity. However, two alternative explanations are possible. First, there could be unobservable differences between long- and short-notice workers. Second, it is possible that a longer notice period reduces a worker's exit probability at later durations. To address these concerns, in Online Appendix F, I relax the assumptions of my model to allow for arbitrary differences between the two groups and for the structural hazards at later durations to vary by notice length up to a certain constant. Although I cannot show that all the parameters of this more general model are identified, I estimate the model by varying the additional parameters and find the values that minimize residuals.⁵ The estimated values that minimize the residuals suggest no mean differences between the two groups. However, they do imply a higher structural hazard for long-notice workers, even beyond the initial 12 weeks. This suggests that the baseline estimates might be underestimating the extent of heterogeneity. Overall, the evidence that heterogeneity across workers plays a predominant role in determining the evolution of the observed exit rate is robust to alternative identifying assumptions.

This paper contributes to the extensive literature on the dynamics of job-finding over the spell of unemployment. Previous empirical studies utilizing the Mixed Hazard model have had to make strong functional form assumptions due to challenges with estimation. Consequently, the evidence on structural duration dependence from these studies is mixed, as highlighted by Machin and Manning (1999) in their review. Recently, Alvarez et al. (2016) revived this strand of work by estimating a Mixed Hitting-Time (MHT) model (Abbring, 2012) using Austrian social security data. They focus on

⁵I verify that the numerical error function is locally convex in all cases.

a selected sample of workers with multiple unemployment spells and are able to estimate the extent of heterogeneity across workers that is fixed between spells. A relative advantage of my approach is that it captures spell-specific heterogeneity.⁶ Another related study, [Mueller et al. \(2021\)](#), utilizes variation in expectations about job-finding from survey data to pin-down variation in actual job-finding rates. While both of these studies also document substantial heterogeneity across jobseekers, my estimator for structural duration dependence is flexible enough to capture changes around UI exhaustion.⁷

Given the challenges with estimating structural duration dependence, researchers have instead focused on estimating its determinants. [Kroft et al. \(2013\)](#) conduct an audit study and find that the likelihood of receiving a callback for an interview declines with the duration of unemployment. However, they note that since they cannot measure worker behavior or employers' ultimate hiring decisions, their estimates only shed light on one determinant of structural duration dependence.⁸ Several papers have also documented how search effort or reservation wages evolve over the spell of unemployment ([Krueger and Mueller, 2011](#); [Marinescu and Skandalis, 2021](#); [DellaVigna et al., 2021](#)). The evidence provided in this paper suggests that, while call-back rates or other factors affecting returns to search matter initially, a worker's optimizing behavior determines the likelihood of exiting unemployment at later durations.

Finally, a substantial body of literature highlights a spike in exit rates at UI exhaustion, where exit rates increase until benefit exhaustion and decline thereafter.⁹ While

⁶For instance, a worker's savings or UI eligibility may change over the months or years by the time this worker becomes unemployed again.

⁷[Alvarez et al. \(2016\)](#) utilizes an optimal-stopping model; a worker finds a job at an optimal stopping time when a Brownian motion with drift hits a barrier. Their model generates an inverse Gaussian distribution of duration for each worker. [Mueller et al. \(2021\)](#) restrict the structural hazard to be monotonic over the spell of unemployment, and their estimator yields a practically flat hazard.

⁸Using a structural model, [Jarosch and Pilossoph \(2019\)](#) argue that if employers statistically discriminate against those with longer durations, then a decline in callback rates only has a marginal effect on workers' exit rates.

⁹[Katz and Meyer \(1990\)](#) first documented the spike in exit rates at benefit exhaustion in the context

the initial increase is consistent with standard search theory, the subsequent decline is not. My estimates reproduce the increase in individual exit probabilities leading up to UI exhaustion but do not find evidence of a decline thereafter. [Boone and van Ours \(2012\)](#) propose storable job offers as an explanation for the spike, while [DellaVigna et al. \(2017\)](#) argue that search models incorporating reference dependence predict a decrease in search effort after benefit exhaustion. My estimates suggest that the decline in the exit rate after UI exhaustion can be attributed to a shift in the composition of surviving workers, as a significant proportion of workers exit unemployment right at benefit exhaustion. However, the individual exit probability remains constant, consistent with the predictions of standard search models.

II CONTEXT AND DATA

In this section, I describe the institutional setting and the data and document how the exit rate out of unemployment varies with the length of notice.

II.A Institutional Details

Under certain circumstances, US employers are required to give notice of layoff. The federal WARN Act mandates that employers with 100 or more full-time employees provide a 60-day advance notice for plant closings and mass layoffs. A plant closing is defined as the shutdown of a site or units within it that results in 50 or more employees losing their jobs within a 30-day period, while a mass layoff is the loss of employment for 500 or more employees during a 30-day period, or 50-499 employees if they constitute one-third or more of the employer's active workforce. The law only applies to layoffs exceeding six months, excluding discharges for cause, voluntary departures, or

of the US. Some recent papers that document this pattern using administrative data are [DellaVigna et al. \(2017\)](#) (Hungary), [Ganong and Noel \(2019\)](#) (US), and [Marinescu and Skandalis \(2021\)](#) (France).

retirements. Some states, such as California, New York, and Illinois, have implemented their own WARN laws that expand the coverage of employment losses beyond what the federal law requires.¹⁰

When it comes to unemployment insurance (UI), US workers who are terminated without cause are typically eligible to receive benefits for a limited duration. Although the UI program is a federal program, each state sets its own benefit levels and durations. Eligibility and benefits may depend on a combination of earnings, hours worked, or weeks worked during a base period, depending on the specific rules of the state's UI program. Typically this base period consists of the first four out of five completed calendar quarters preceding the claim filing date. In most states, the maximum period for receiving benefits is 26 weeks. Nine states have a uniform benefit duration of 26 weeks, while the benefit durations in the remaining states vary depending on the applicant's earnings history. Additionally, a program for extended benefits has been in place since a 1970 amendment to the Federal Unemployment Tax Act (FUTA), which can be triggered by the state unemployment rate. Temporary programs have also been implemented to extend benefits during recessions.

II.B Data Description and Sample Construction

I use data from the Displaced Worker Supplement (DWS) for the years 1996-2020. DWS is fielded biennially along with the basic monthly Current Population Survey (CPS) in January or February. The survey is administered to individuals who report having *lost or left* a job within the past three years due to a plant closure, their position being abolished, or having insufficient work at their previous employment. Apart from details on workers' lost and current jobs, DWS also collects the length of the notice period workers received before being laid off and the length of time they took to find another

¹⁰It is not possible to exploit policy variation across states, say in a differences-in-difference framework, due to confounding pre-trends; both California and New York implemented these laws in the aftermath of a national recession.

job.

For my analysis, I consider individuals aged 21 to 64 years old, who worked full-time for at least six months and were provided health insurance at their lost job. I exclude individuals who expected to be recalled and those whose lost job was self-employment. I also exclude individuals who did not receive any notice at all because it is uncertain whether they were displaced or quit their jobs voluntarily.¹¹ Lastly, I exclude individuals who experienced job loss in the preceding calendar year. The reason for this exclusion is that we do not observe completed unemployment spells for workers who haven't found new employment by the time of the survey. However, for the sample of workers who lost their jobs at least one year prior, we can calculate the exit rate out of unemployment at all durations less than a year. See Section C.1 in the Online Appendix for additional details on data construction. Table C.1 compares the characteristics of the workers in my sample to all workers in the CPS and DWS. Overall, workers in my sample are slightly older and more educated.

In the DWS, workers who received a notice report whether the length of their notice was less than one month, between one to two months, or greater than two months. Since there is a negligible difference in the exit rates for the first two categories, I combine them together into a single category referred to as *short notice*. Meanwhile, a notice length exceeding two months is categorized as *long notice*. Table 1 presents the summary statistics separately for workers with short and long notice in the sample. Columns (1) and (2) display the raw averages for the sample, revealing notable differences between the two groups. Workers with longer notice tend to be older, more likely to be female, and less likely to be Black. Additionally, workers laid off during plant closures are more likely to receive longer notice, potentially due to compliance

¹¹While the DWS aims to capture separations due to firms facing economic challenges, the distinction between quits and voluntary layoffs is blurred. Firms facing economic challenges may reduce hours or wages instead of laying off workers, which can prompt workers, especially those with better alternatives, to quit voluntarily (Farber, 2017).

TABLE 1: DESCRIPTIVES BY NOTICE LENGTH

	Unbalanced			Balanced		
	Short (1)	Long (2)	Diff. (2)-(1)	Short (3)	Long (4)	Diff. (4)-(3)
Age	42.24 (0.23)	43.85 (0.27)	1.61*** (0.35)	43.03 (0.22)	42.97 (0.28)	-0.06 (0.36)
Female	0.43 (0.01)	0.46 (0.01)	0.04** (0.02)	0.45 (0.01)	0.46 (0.01)	0.01 (0.02)
Married	0.59 (0.01)	0.65 (0.01)	0.05*** (0.02)	0.61 (0.01)	0.61 (0.01)	-0.01 (0.02)
Black	0.10 (0.01)	0.08 (0.01)	-0.02** (0.01)	0.09 (0.01)	0.09 (0.01)	0.00 (0.01)
College Degree	0.39 (0.01)	0.38 (0.01)	0.00 (0.02)	0.39 (0.01)	0.40 (0.01)	0.01 (0.02)
Plant Closure	0.40 (0.01)	0.63 (0.01)	0.23*** (0.02)	0.49 (0.01)	0.49 (0.01)	-0.01 (0.02)
Union Membership	0.15 (0.01)	0.15 (0.01)	0.00 (0.01)	0.15 (0.01)	0.16 (0.01)	0.00 (0.01)
In Metro Area	0.83 (0.01)	0.82 (0.01)	-0.01 (0.01)	0.83 (0.01)	0.83 (0.01)	0.00 (0.01)
Years of Tenure	6.53 (0.14)	9.22 (0.20)	2.69*** (0.24)	7.74 (0.16)	7.78 (0.18)	0.03 (0.24)
Log Earnings	6.50 (0.01)	6.56 (0.01)	0.05*** (0.02)	6.53 (0.01)	6.53 (0.02)	-0.01 (0.02)
Observations	2147	1409		2147	1409	

Note: The sample consists of respondents from the Displaced Worker Supplement (DWS) for the years 1996-2020, who were between ages 21 to 64, had worked full-time for at least six months at their previous job, received health insurance from their former employer, and did not expect to be recalled. The sample excludes workers who were laid off in the year immediately preceding the survey. Short notice refers to a notice period of less than a month or between one and two months, while long notice refers to a notice period exceeding two months. Columns (1) and (2) present raw averages for the sample, while columns (3) and (4) show weighted averages, where the weights correspond to the inverse of the estimated probabilities of receiving short or long notice.

with the WARN law. Workers with longer notice also tend to have longer job tenure and higher earnings at their previous job.

To isolate the impact of notice from these additional correlates, which may affect the probability of exiting unemployment, I reweight the sample using inverse propensity score weighting. I use a logistic regression model to predict the likelihood of receiving a longer notice based on several covariates. These covariates consist of age, gender, marital status, race, education, location characteristics, the reason for displacement, year of displacement, industry and occupation of the lost job, as well as union status, tenure, and earnings at the lost job. I then utilize the propensity scores to assign weights to the observations. Specifically, individuals with the long notice are assigned a weight of $1/\hat{p}(X_i)$, where $\hat{p}(X_i)$ is the estimated probability of receiving the long notice from the regression model for an individual with covariates X_i . On the other hand, individuals who received the short notice are assigned a weight of $1/(1 - \hat{p}(X_i))$.

The summary statistics for the reweighted sample are presented in columns (3) and (4) of Table 1. After reweighting, the observable differences between the two groups disappear, indicating that the weights effectively minimize the observed disparities. Section C.2 in the Online Appendix provides additional details on propensity score estimation. Figure C.1 demonstrates a high degree of overlap in the estimated propensity score distributions between long and short-notice workers. Additionally, Figures C.2 and C.3 depict the balance of the weighted sample with respect to the displacement year and industrial and occupational composition, respectively.

II.C Distribution of Unemployment Duration

In this section, I explore how a longer notice impacts the exit rate over the spell of unemployment. Workers who receive a layoff notice may start searching for a job before separating from their previous employer. In this case, it is possible that some of these workers may secure a new job during the notice period, thus avoiding any period of

TABLE 2: OBSERVED EXIT RATE – EARLY IN THE SPELL

	(1)	(2)	(3)	(4)
<u>PANEL A. $\mathbb{I}\{\text{UNEMPLOYMENT DURATION} = 0 \text{ WEEKS}\}$</u>				
>2 month notice	0.112*** (0.013)	0.087*** (0.015)	0.087*** (0.017)	0.085*** (0.014)
<u>PANEL B. $\mathbb{I}\{\text{UNEMPLOYMENT DURATION} \leq 12 \text{ WEEKS}\}$</u>				
>2 month notice	0.091*** (0.017)	0.082*** (0.018)	0.074*** (0.020)	0.074*** (0.018)
Controls	No	Yes	No	Yes
Weights	No	No	Yes	Yes
	3556	3556	3556	3556

Note: The table presents estimates from linear regression models, where the main independent variable is an indicator variable that takes a value of 1 if the individual received a notice of more than 2 months, and 0 if they received a notice of less than 1 month or between 1-2 months. The dependent variable is an indicator for reporting an unemployment duration of 0 weeks (Panel A) or less than 12 weeks (Panel B). The weights are generated using inverse probability weighting (IPW). Robust standard errors are reported in the parenthesis.

unemployment. In the data, 12.4% of the workers with the short layoff notice report no duration of unemployment. Since workers with longer notice periods have more time to search for a new job while still employed, we expect their chances of avoiding unemployment to be even greater.

In Table 2, panel A, I examine the relationship between receiving a long notice and reporting an unemployment duration of 0. Columns (1) and (2) present estimates from unweighted regressions, while columns (3) and (4) present weighted regression estimates using the weights described in the previous section. Additionally, columns (2) and (4) include a comprehensive set of controls identical to the ones used to generate the weights. The table shows that the impact of a lengthier notice on the exit probability is reduced after accounting for observable characteristics of the separation. The coefficient in column (2) indicates that individuals who receive a longer notice are 8.8

percentage points more likely to avoid unemployment. Similar estimates are observed in columns (3) and (4) as well. Notably, the inclusion of controls in column (4) does not lead to a change in the coefficient, indicating that the weighting has effectively achieved balance in terms of the covariates across the two groups. In panel B of Table 2, I present a similar regression analysis, but this time using an indicator for exiting unemployment within the first 12 weeks. The results show that the exit rate out of unemployment is 7.4 percentage points higher for long-notice workers compared to the short-notice group.

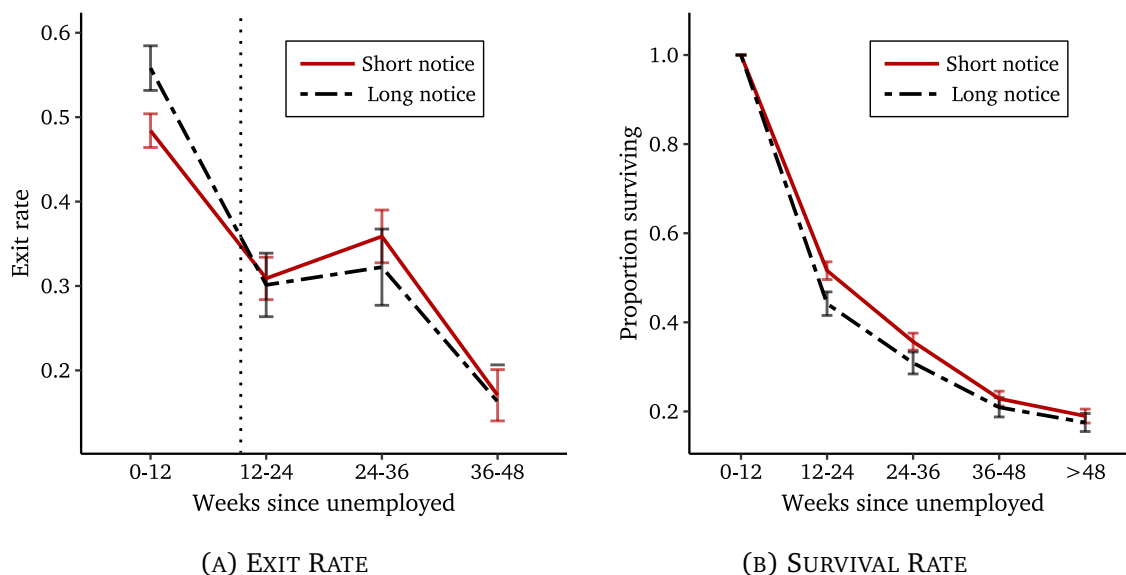
To examine how the exit rate varies with the length of notice over the spell of unemployment, I bin unemployment duration into 12-week intervals.¹² Figure 1 presents the exit rate and the survival rate separately for the long- and short-notice workers over the spell of unemployment. Note that the rates are calculated using the weighted sample to ensure that the comparison is between similar groups of workers who received different lengths of notice.¹³ Approximately 56% of individuals with a long notice exit within the first 12 weeks, while only 49% of those with a short notice do the same. However, over the course of unemployment, individuals with shorter notice periods catch up, resulting in almost identical survival rates for both groups by the 48th week of unemployment. As shown in panel A of Figure 1, for all durations beyond 12 weeks, individuals with shorter notice periods have a higher exit rate compared to those with longer notice periods.

I interpret the higher exit rate for short-notice workers beyond the initial 12 weeks as evidence for heterogeneity across workers. When workers are heterogeneous, those with better chances of exiting unemployment do so earlier. Given that a larger propor-

¹²See Figure C.6 in the Online Appendix for the presentation of data with alternative binning definitions.

¹³Online Appendix E presents the unweighted exit rates and corresponding estimates obtained from the Mixed Hazard model. It also presents data and estimates for a subset of the sample by excluding observations with less than one month's notice, thereby only comparing workers with 1-2 months' notice and those with more than 2 months' notice.

FIGURE 1: EXIT AND SURVIVAL RATE – LATER IN THE SPELL



Note: Short notice refers to a notice of less than 2 months, and long notice refers to a notice of more than 2 months. Panel A presents the weighted proportion of individuals exiting unemployment in each interval amongst those who were still unemployed at the beginning of the interval. Panel B presents the weighted proportion of individuals who are unemployed at the beginning of each interval. Error bars represent 90% confidence intervals.

tion of long-notice workers exit earlier, the long-notice group will have a lower proportion of individuals with higher exit probabilities, which is reflected in the (average) exit rate. It is important to note that this interpretation holds under the condition that longer notice does not directly reduce the probability of exiting unemployment at later durations. In the following section, I formally outline the assumptions necessary to identify heterogeneity and duration dependence in a Mixed Hazard model, and I also discuss the plausibility of these assumptions and potential violations.

III ECONOMETRIC FRAMEWORK

This section illustrates how variation in notice length can be used to identify structural duration dependence. Specifically, I set up a Mixed Hazard model in discrete time and specify the assumptions under which the key components of this model are

identified. The model and assumptions are outlined in Section III.A, while the main identification result is presented in Section III.B. The intuition behind identification is explained in III.B. All proofs are presented in Appendix A. Section D.2 in the Online Appendix presents an extension to deal with right-censored data.

III.A Mixed Hazard Model in Discrete Time

The realized unemployment duration, denoted by the random variable D , takes values in $\{1, 2, 3, \dots\}$. The cumulative and probability distribution functions of unemployment duration are denoted by $G(\cdot)$ and $g(\cdot)$, respectively. Workers are heterogeneous and have an unobservable fixed type ν with the cumulative distribution $F(\cdot)$. Before being laid off, workers receive a notice period of length L , where L takes on discrete values. Additionally, X denotes a vector of observable pre-notice characteristics of the layoff. These characteristics may include details about the job, the worker, or the circumstances of the layoff. The distribution of X is denoted by $F_X(\cdot)$.

The hazard $h(d|\nu, l, X)$ represents an individual's probability of exiting unemployment at duration d , given that the individual has not exited yet. According to the Mixed Hazard model, this probability can be expressed as the product of the individual's unobserved type ν and a structural component $\psi_l(d, X)$.¹⁴

Assumption 1. (*Mixed Hazard*) An individual's exit probability at duration d is given by:

$$h(d|\nu, l, X) = \psi_l(d, X)\nu$$

where the structural hazard $\psi_l(d, X) \in (0, \infty)$ and worker's type $\nu \in (0, \bar{\nu}]$ with $\bar{\nu} = 1 / \max_{d, l, X} \{\psi_l(d, X)\}$.

¹⁴Lancaster (1979) expanded the proportional hazard model (Cox, 1972) to incorporate unobserved heterogeneity. His Mixed Proportional Hazard (MPH) model represented the hazard rate as a product of a regression function, a structural hazard that varies with duration, and the worker's unobserved type. The Mixed Hazard model formulated here is similar to Lancaster's MPH model, but permits non-proportional effects of observable characteristics and distinguishes the length of notice from other observed variables.

The structural hazard $\psi_l(d, X)$ is common to all individuals with observable characteristics X and a notice period of length l , but it varies with the duration of unemployment. The restrictions on the structural hazard and the support of ν in Assumption 1 guarantee that individual exit probabilities lie between 0 and 1.

Identifying the structural hazard $\psi_l(d, X)$ is challenging because we only observe when each worker exits unemployment, but not the underlying exit probability for individual workers at all durations. Although the duration distribution allows us to compute the exit rate at each duration, it only captures the average hazard for those who have survived up to that point. Since low-type workers are more likely to survive until later durations, the observed exit rate decreases more with duration than the underlying individual exit probabilities. This is still the case even after controlling for observable characteristics due to unobserved heterogeneity.¹⁵ The following proposition formally states this result.

Proposition 1. *Under Assumption 1, the exit rate at duration d , denoted by $\tilde{h}(d|l, X)$, can be expressed as follows:*

$$\tilde{h}(d|l, X) = \frac{g(d|l, X)}{1 - G(d-1|l, X)} = \psi_l(d, X) \mathbb{E}(\nu | D \geq d, l, X)$$

Moreover, the average type of workers who survive until d , $\mathbb{E}(\nu | D \geq d, l, X)$, decreases with the unemployment duration d .

The proposition above highlights that the exit rate $\tilde{h}(d|l, X)$ is impacted by both the structural duration dependence $\psi_l(d, X)$ and the changing worker composition over the unemployment spell captured by $\mathbb{E}(\nu | D \geq d, l, X)$. If the observed exit rate $\tilde{h}(d|l, X)$ declines over the spell of unemployment, it is not possible to distinguish between the scenario where there is no structural duration dependence, but significant worker heterogeneity causes the average type of workers and the observed exit rate to decline,

¹⁵The exit rate in the data declines even after controlling for a rich set of observables. See Figure C.7 in the Online Appendix.

and the scenario where there is no worker heterogeneity, but the structural hazard declines over the spell of unemployment. Both of these scenarios would be consistent with the observed decline in $\tilde{h}(d|l, X)$, and hence, structural duration dependence is not identified in the model formulated so far.

I now introduce two additional assumptions under which variation in notice length leads to the identification of structural duration dependence. The first assumption is conditional independence, which states that the length of notice is independent of the worker's unobservable type given observable characteristics. In other words, for workers with similar observable characteristics, there is no systematic difference in the length of notice given to workers with different unobservable types.¹⁶

Assumption 2. (*Conditional Independence*) *The length of notice L is independent of the worker's unobservable type v , given observable characteristics X , i.e., $L \perp v|X$.*

The second assumption, referred to as stationarity, states that the length of notice does not affect an individual's exit probability after the first period.

Assumption 3. (*Stationarity*) *For all l, X , and $d > 1$,*

$$\psi_l(d, X) = \psi(d, X)$$

The rationale for Assumption 3 is that workers with longer notice periods have more time to search for a new job before separating from their previous employer, potentially increasing their likelihood of finding a job at the beginning of their unemployment spell. However, if duration dependence in job-finding is caused by factors such as human capital depreciation due to prolonged unemployment or employers discriminating against long-term unemployed workers, then a worker's exit probability later in the spell should only vary with the unemployment duration and not with the length of notice received

¹⁶In Online Appendix F, I provide estimates from a model that permits the underlying type distribution to differ across various notice lengths, instead of assuming conditional independence.

at the onset of the spell. Given that I bin unemployment duration in 12-week intervals, this assumption translates to the length of notice only impacting the probability of exit within the first 12 weeks and not thereafter.

More generally, Assumption 3 implies that individual exit probabilities vary only with the duration of unemployment and not with time elapsed since the start of the job search.¹⁷ This assumption would be violated if time spent searching increases or decreases an individual’s likelihood of exiting unemployment. For instance, if workers learn while searching and become better at job search (Burdett and Vishwanath, 1988; Gonzalez and Shi, 2010) then those with longer notice would have a higher hazard even beyond the initial period. On the other hand, time spent searching may decrease the exit probability if workers first apply to all jobs in stock but subsequently only apply to newly posted jobs (Coles and Smith, 1998).¹⁸ While I show in Online Appendix F that the latter is not supported by the data, my estimates will underestimate the extent of unobserved heterogeneity if the former holds.¹⁹

III.B Identification Results

Theorem 1. *Under Assumptions 1–3, for any l, l' with $\psi_l(1, X) \neq \psi_{l'}(1, X)$ and some integer \bar{D} , the structural hazards $\{\psi_l(1, X), \psi_{l'}(1, X), \{\psi(d, X)\}_{d=2}^{\bar{D}}\}$ and the conditional moments of the type distribution $\{\mathbb{E}(v^k | X)\}_{k=1}^{\bar{D}}$ are identified up to a scale from the condi-*

¹⁷This assumption aligns with a large class of search models, including those that involve non-stationarity. For instance, the model proposed by Lentz and Tranæs (2005), in which workers start searching harder over time as their savings run down, would be consistent with this assumption as savings only start depleting once unemployed.

¹⁸Another possibility for why individual exit probability may decline with time spent searching could be that workers get discouraged over time and stop trying. However, individuals who eventually drop out of the labor force are excluded from the analysis.

¹⁹In particular, I estimate a more general model that allows the structural hazards to vary with the length of notice even beyond the initial period. It is not possible to show that all the parameters of this more general model are identified. Instead, I estimate the model by varying the values of additional parameters not included in the baseline model and identifying optimal values that minimize residuals. Importantly, I verify that the numerical error function is locally convex in all cases. The estimated values that minimize the residuals imply a higher structural hazard for long-notice workers, even beyond the initial 12 weeks.

tional duration distribution $\{G(d|l, X), G(d|l', X)\}_{d=1}^{\bar{D}}$.

The above theorem establishes that if the first-period hazard varies for two different notice lengths, we can identify the structural hazards up to \bar{D} and the first \bar{D} moments of ν conditional on X using the conditional duration distributions for both notice lengths up to \bar{D} . A direct implication of this result is that if X does not enter the structural hazard and we assume independence instead of conditional independence, we can identify the model using duration distributions unconditional on X . The following corollary presents this result formally.

Corollary 1. *Assuming Assumption 1, Assumption 3, and $\psi_l(d, X) = \psi_l(d)$ hold, and L is independent of ν i.e. $L \perp \nu$, then for any l, l' , with $\psi_l(1) \neq \psi_{l'}(1)$ and some integer \bar{D} , the structural hazards $\{\psi_l(1), \psi_{l'}(1), \{\psi(d)\}_{d=2}^{\bar{D}}\}$ and the moments of the type distribution $\{\mathbb{E}(\nu^k)\}_{k=1}^{\bar{D}}$ are identified up to a scale from $\{G(d|l), G(d|l')\}_{d=1}^{\bar{D}}$.*

Neither of the two results mentioned above is ideal for application to the data. The first result has a limitation in that $G(d|l, X)$ is only well-defined for discrete values of X , and even then, it may be imprecisely estimated if each bin size is not large enough. On the other hand, the second result allows us to use duration distributions that are unconditional on X , but it imposes a stronger restriction of unconditional independence, which may not hold in the data. To address these limitations, I present an additional result below, which allows controlling for observables more flexibly. Specifically, if observable characteristics enter the structural hazard proportionally, as in the MPH model, the model's parameters are identified under conditional independence using the “weighted” unemployment distribution.²⁰ The weights are chosen to ensure that the weighted distribution of observable characteristics X does not vary by the length of notice.

Before presenting the formal result, I introduce some additional definitions. In par-

²⁰Note that the length of notice still enters the structural hazard non-proportionally.

ticular, the weighted distribution of X using the set of weights ω , denoted by $f_X^\omega(\cdot)$, is defined as: $f_X^\omega(x|L) = f_X(x|L)\omega_L(x)$. Here, the weights are chosen to ensure $f_X^\omega(\cdot)$ is a proper distribution on some support \mathcal{X} . Accordingly, the weighted unemployment duration distribution is denoted by $G^\omega(d|l)$, and is defined as:

$$G^\omega(d|l) = \int_{\mathcal{X}} G(d|l, x) f_X^\omega(x|l) \partial x$$

Finally, define the k 'th weighted moment of ν as $\mu_k^\omega = \int_{\mathcal{X}} \phi(x)^k \mathbb{E}(\nu^k|x) f_X^\omega(x|l) \partial x$.

When the weights $\omega_L(x)$ are selected to ensure that the resulting weighted distribution of X does not vary by L , i.e., $F_X^\omega(x|L) = F_X^\omega(x)$, then the weighted unemployment distribution $G^\omega(d|L)$ reflects how the unemployment duration varies with the length of notice while taking observable differences across notice lengths into account. Consequently, the following result asserts that the model's parameters can be determined from the weighted duration distributions.

Proposition 2. *Suppose Assumptions 1–3 and $\psi_l(d, X) = \psi_l(d)\phi(X)$ hold. For any l, l' with $\psi_l(1) \neq \psi_{l'}(1)$, consider the set of weights $\omega_l(x)$ and $\omega_{l'}(x)$ that ensure $f_X^\omega(x) = f_X(x|l) = f_X(x|l')$ for all x on some common support \mathcal{X} of $f_X(\cdot|l)$ and $f_X(\cdot|l')$. Then, the structural hazards $\{\psi_l(1), \psi_{l'}(1), \psi(d)\}_{d=2}^{\bar{D}}$ and the weighted moments of the type distribution $\{\mu_k^\omega\}_{k=1}^{\bar{D}}$ are identified up to a scale from the weighted unemployment distribution $\{G^\omega(d|l), G^\omega(d|l')\}_{d=1}^{\bar{D}}$.*

The main results in the paper correspond to the estimation strategy implied by Proposition 2. Additionally, Section D.2 in the Online Appendix provides an extension to incorporate right-censored data. In particular, the result shows that if the censoring time is independent of notice length, we can restrict the sample to individuals who were censored after \bar{D} , and identify structural duration hazards upto \bar{D} .

III.C Intuition for Identification

In this section, I elucidate the intuition behind the identification result. To simplify the explanation, I focus on the case without observable characteristics, as incorporating them does not provide any additional insights regarding identification. In this model, an individual worker's exit probability is given by $h(d|l, \nu) = \psi_l(d)\nu$, and ν is independent of L . Note that independence implies $f(\nu|L) = f(\nu)$. For brevity, let us denote the first and second moments of ν by $\mu_1 = \mathbb{E}(\nu)$ and $\mu_2 = \mathbb{E}(\nu^2)$, respectively. It is worth noting that the variance of ν , given by $\text{var}(\nu) = \mu_2 - \mu_1^2$, captures the extent of heterogeneity across workers.

To see why the identification result holds, note that the exit rate in the first period is given by $\tilde{h}(1|l) = g(1|l) = \psi_l(1)\mu_1$. Since $\tilde{h}(2|l) = g(2|l)/(1 - g(1|l))$, we can write the exit rate at $d = 2$ as

$$\tilde{h}(2|l) = \psi(2) \left(\frac{\mu_1 - \psi_l(1)\mu_2}{1 - \psi_l(1)\mu_1} \right) = \psi(2)\mu_1 \left(\frac{1 - \tilde{h}(1|l)(\mu_2/\mu_1^2)}{1 - \tilde{h}(1|l)} \right)$$

The second equality in the above equation follows from $\psi_l(1) = \tilde{h}(1|l)/\mu_1$. In the presence of heterogeneity, the variance of ν is greater than zero, which means that $\mu_2/\mu_1^2 > 1$. Therefore, based on the expressions for $\tilde{h}(1|l)$ and $\tilde{h}(2|l)$, we can observe that $\tilde{h}(2|l)/\tilde{h}(1|l)$ will always be smaller than $\psi(2)/\psi_l(1)$. Furthermore, the greater the variance of ν (i.e., the more heterogeneity across workers), the larger μ_2/μ_1^2 will be, and the more distant $\tilde{h}(2|l)/\tilde{h}(1|l)$ will be from $\psi(2)/\psi_l(1)$. This occurs because greater heterogeneity across workers implies that the composition of workers from the first to the second period changes more drastically. For instance, in the absence of heterogeneity across workers where $\mu_2/\mu_1^2 = 1$, the composition across both periods is unchanged, and thus $\tilde{h}(2|l)/\tilde{h}(1|l) = \psi(2)/\psi_l(1)$.

If we knew the extent of heterogeneity across workers as captured by μ_2/μ_1^2 , we could determine how the composition changes from the first to the second period and estimate the structural duration dependence $\psi(2)/\psi_l(1)$ from the observed duration

dependence $\tilde{h}(2|l)/\tilde{h}(1|l)$. The variation in notice lengths allows us to learn about the underlying heterogeneity and estimate structural duration dependence. To understand why this is the case, note that for two lengths of notice l and l' , the following expression holds:

$$\frac{\tilde{h}(2|l)}{\tilde{h}(2|l')} = \left(\frac{1 - \tilde{h}(1|l)(\mu_2/\mu_1^2)}{1 - \tilde{h}(1|l)} \right) / \left(\frac{1 - \tilde{h}(1|l')(\mu_2/\mu_1^2)}{1 - \tilde{h}(1|l')} \right)$$

Assuming without loss of generality that $\tilde{h}(1|l') > \tilde{h}(1|l)$, we can see from the above expression that then $\tilde{h}(2|l)/\tilde{h}(2|l') \geq 1$. This is because more individuals with notice length l' leave in the first period, leading to a worse composition for that group in the second period. Furthermore, when the variance across workers is higher, $\tilde{h}(2|l)$ will be further above $\tilde{h}(2|l')$. Thus, the difference in exit rates among workers with different notice lengths provides information about the degree of heterogeneity, and we can use the above expression to compute μ_2/μ_1^2 . Once we know μ_2/μ_1^2 , we can plug that back into the expression for $\tilde{h}(2|l)/\tilde{h}(1|l)$ and estimate the structural duration dependence $\psi(2)/\psi_1(1)$. In summary, the difference in exit rates at duration $d = 2$ across notice lengths reflects differences in the composition of remaining workers. Therefore, comparing exit rates of workers with different notice lengths can provide insights into the extent to which underlying heterogeneity impacts exit rates. A similar argument applies to identifying structural hazards beyond the second period.²¹

IV ESTIMATION

Generalized Method of Moments (GMM). Using the identification result presented in Proposition 2, we can use the Generalized Method of Moments (GMM) to construct

²¹To understand why higher moments determine the hazard at later durations, we can consider how the composition of workers changes from $d = 2$ to $d = 3$. This change depends on the level of heterogeneity across workers at the start of $d = 2$. If the distribution of heterogeneity has a positive skew, the variance among individuals who survive to $d = 2$ would be lower than that among individuals at the start of $d = 1$. This is because the few individuals with a high likelihood of exiting unemployment would have already left, reducing the variance among surviving workers.

a consistent estimator for the structural hazards and weighted moments of the unobserved heterogeneity distribution. Since the model is only identified up to scale, I normalize the first weighted moment to $\mu_1^\omega = 1$. With J possible notice lengths, the vector of unknown parameters is given by $\Theta = \{\{\psi_l(1)\}_{l=1}^J, \{\psi(d)\}_{d=2}^{\bar{D}}, \{\mu_k^\omega\}_{k=2}^{\bar{D}}\}$ and has a total of $2(\bar{D} - 1) + J$ unknown parameters.

Now, for each individual i , let us define the following moment condition:

$$m_i(l, d, \Theta) = \mathbb{I}\{L_i = l\} w_i [\mathbb{I}\{D_i = d\} - g^w(d|l; \Theta)]$$

Here, $g^w(d|l; \Theta)$ represents a function of the parameters as implied by the model under the assumptions for Proposition 2. The weights w_i depend on X and l and ensure that the distribution of observables is similar across individuals with different notice lengths.²² We can now stack moment conditions pertaining to different notice lengths and durations in one vector, denoted by $m_i(\Theta) = \{\{m_i(l, d, \Theta)\}_{d=1}^{\bar{D}}\}_{l=1}^J$. Under the model assumptions, we have $\mathbb{E}[m_i(\Theta)] = 0$.²³ Also, note that $m_i(\Theta)$ contains $J \times \bar{D}$ moment conditions, and as shown in Proposition 2, our parameters of interest are identified from these moment conditions as long as $J > 1$.

To construct the GMM estimator, note that the corresponding sample average for $\mathbb{E}[m_i(\Theta)]$ can be written as:

$$\hat{m}(\Theta) = \frac{1}{n} \sum_{i=1}^n m_i(\Theta) = \left\{ \left\{ \pi_l [\hat{g}^\omega(d|l) - g^\omega(d|l; \Theta)] \right\}_{d=1}^{\bar{D}} \right\}_{l=1}^J$$

Here, n is the sample size, $\hat{g}^\omega(d|l) = \left(\sum_{i:L_i=l} w_i \mathbb{I}\{D_i = d\} \right) / \left(\sum_{i:L_i=l} w_i \right)$ is the sample counterpart of the weighted duration distribution, and $\pi_l = \left(\sum_{L_i=l} w_i \right) / n$.

The GMM estimator $\hat{\Theta}$ is then given by: $\hat{\Theta} = \arg \max_{\Theta} \hat{m}(\Theta)' \hat{W} \hat{m}(\Theta)$. When the

²²Note that in principle, there is also uncertainty associated with the selected weights (Abadie and Imbens, 2016). However, here I ignore this first-step uncertainty while deriving the distribution of my estimator.

²³See Appendix Section A.6 for the proof.

model is just-identified, \hat{W} is given by the identity matrix. In the case of over-identification, the efficient weighting matrix is given by $\hat{W} = \hat{\Omega}^{-1}$, where $\hat{\Omega} = \left[\frac{1}{n} \sum_{i=1}^n m_i(\hat{\Theta}) m_i(\hat{\Theta})' \right]^{-1}$. Using the two-step estimation process, we can compute $\hat{\Theta}$. The asymptotic distribution of this estimator is given by $\sqrt{n}(\hat{\Theta} - \Theta) \rightarrow N(0, (\hat{M}' \hat{\Omega}^{-1} \hat{M})^{-1})$, where $\hat{M} = \partial \hat{m}(\hat{\Theta}) / \partial \Theta$.

²⁴

Functional Form for Structural Hazard. Even though the model is identified non-parametrically, given small sample sizes, to minimize the number of estimated parameters, I assume that the structural hazard $\psi(d)$ for $d > 1$ has a log-logistic form as follows

$$\psi(d) = \frac{(\alpha_2/\alpha_1)(d/\alpha_1)^{\alpha_2-1}}{1 + (d/\alpha_1)^{\alpha_2}} \quad (1)$$

where $\alpha_1 > 0, \alpha_2 > 0$. The hazard function in equation (1) is monotonically decreasing when $\alpha_2 \leq 1$ and is unimodal, initially increasing and subsequently decreasing when $\alpha_2 > 1$. The mode or the turning point is $\alpha_1(\alpha_2 - 1)^{1/\alpha_2}$.²⁵

V DURATION DEPENDENCE AND HETEROGENEITY

VA *Baseline Estimates*

Table 3 presents the main estimates from the Mixed Hazard model. Since I normalized the weighted mean in the first period to equal 1, the estimated structural hazards corresponding to the first period for short and long-notice individuals coincide with their corresponding observed exit rates in the data. The last two lines in panel A of Table 3 show the estimated parameters for the log-logistic function specified in equation

²⁴We can construct a GMM estimator using data from right-censored spells in a similar manner. In particular, the sample moments for estimation will now pertain to the distribution of observed durations conditional on the censoring time greater than some \bar{D} .

²⁵This provides a flexible parametrization for the structural hazard relative to other commonly used parametrization, such as Weibull or Gompertz, as it allows the structural hazard to be non-monotonic. However, I also present estimates with alternative functional form restrictions and non-parametric estimates in Online Appendix E.

TABLE 3: ESTIMATION RESULTS

Parameter	Explanation	Estimate	SE
<i>Panel A: Estimated Parameters</i>			
$\psi_S(1)$	Structural hazard 0-12 weeks: Short notice	0.49	0.01
$\psi_L(1)$	Structural hazard 0-12 weeks: Long notice	0.55	0.01
α_1	Scale parameter for $\psi(d)$	1.21	0.09
α_2	Shape parameter for $\psi(d)$	1.46	0.45
<i>Panel B: Duration Dependence</i>			
$\bar{\psi}(1)$	Structural hazard: 0-12 weeks	0.52	0.01
$\psi(2)$	Structural hazard: 12-24 weeks	0.40	0.05
$\psi(3)$	Structural hazard: 24-36 weeks	0.61	0.08
$\psi(4)$	Structural hazard: 36-48 weeks	0.63	0.09
<i>Hansen-Sargan Test</i>			
Test statistic: 2.14		Critical value, $df = 1, \chi_{0.05}^2$: 3.84	

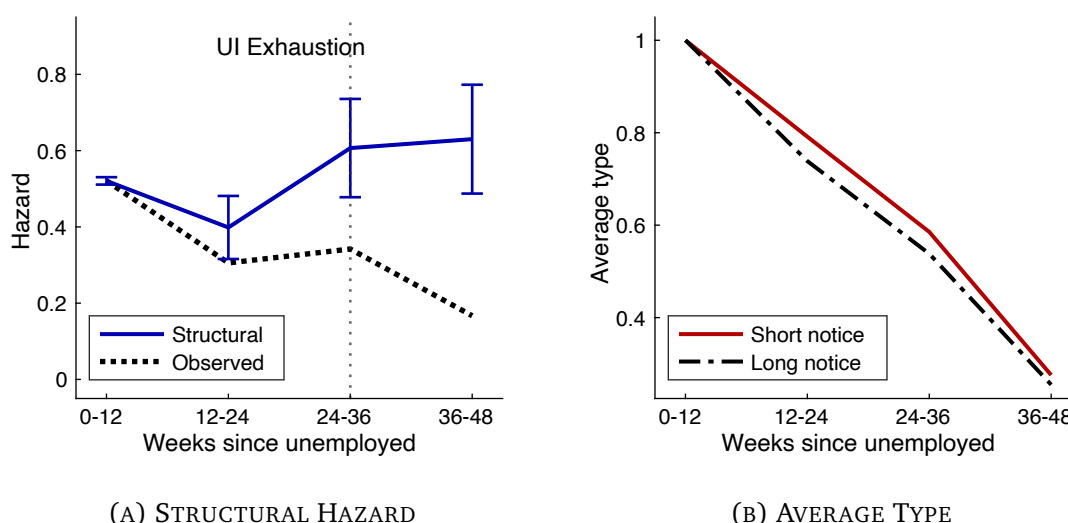
Note: The table presents estimates from the Mixed Hazard model. The first weighted moment is normalized to one, and structural duration dependence is specified by equation (1). Panel A shows the estimated parameters from the model, and panel B presents structural hazards implied by the estimated parameters. The standard errors for the structural hazards are calculated using the delta method.

(1) used to model structural dependence.

The structural hazards implied by these parameters are presented in panel B of Table 3 and panel A of Figure 2. Additionally, panel A of Figure 2, alongside the estimated hazard, also shows the observed exit rate from the data, averaged across workers with short and long-notice. This figure shows that the estimated hazard consistently exceeds the observed hazard throughout the unemployment spell, indicating the role of underlying heterogeneity. While the observed hazard in the data declines by 41% over the first 24 weeks, the estimated structural hazard only decreases by 23% during the same period. Hence, accounting for heterogeneity suggests that only half of the observed decline in the first 24 weeks can be attributed to structural duration dependence.

Moreover, the estimated structural hazard increases by 52% from 12-24 to 24-36 weeks, a more pronounced increase than the observed hazard. This pattern possibly

FIGURE 2: BASELINE ESTIMATES



Note: Solid line in panel A presents estimates for structural hazards as implied by the estimated parameters in panel A of Table 3. The dotted line in panel A presents the observed exit rate from the data, averaged across workers with short and long notice. Panel B presents the implied average type at each duration for those with short and long notice. Error bars represent 90% confidence intervals.

reflects individuals approaching the exhaustion of their unemployment insurance (UI) benefits.²⁶ As previously noted, there is variation across individuals in the eligible duration of UI receipt. However, a significant proportion of individuals are eligible for UI benefits that last for 26 weeks, which coincides with the third interval. Figure C.4 in the Online Appendix shows that the proportion of individuals reporting exhausting their UI benefits jumps by 20 percentage points at 26 weeks.²⁷ This finding of increasing structural hazard leading up to benefit exhaustion is consistent with individuals intensifying their job search efforts or lowering their expectations to secure employment before de-

²⁶Since the sample consists of displaced workers, a substantial portion of these individuals should be eligible for UI benefits. Table C.3 in the Online Appendix shows that 80% of individuals unemployed for longer than 12 weeks report receiving UI benefits in my sample.

²⁷On two occasions during the sample period, following the 2000 and 2008 recession, there was a widespread extension of UI benefits beyond the standard 26 weeks. As a robustness check, I estimate the model separately for individuals laid off during these periods of extended benefits. Figure C.5 confirms that the selected sample had fewer individuals who reported exhausting their benefits within 24-36 weeks. The results from this estimation exercise, presented in Figure E.5, indicate a slightly elevated estimate for the structural hazard even beyond the third interval.

pleting their benefits. Finally, while the observed hazard continues declining even after 36 weeks, the structural hazard remains constant.

Panel B of Figure 2 displays the average type implied by the model for individuals with short and long notice periods throughout the unemployment spell. For both groups, the average type deteriorates over the course of unemployment. However, for individuals with longer notice periods, the composition worsens more between 0-12 and 12-24 weeks, indicating a higher exit rate in the initial period for this group. By the end of 36 weeks, when a significant number of individuals have already left unemployment, there is little difference in the average type between the two groups.

Overall, the estimated pattern of structural duration dependence aligns with existing evidence from audit studies on call-back rates and with the predictions of search theory. I find that individual exit probabilities decline during the first 5 months, which can be attributed to duration-based employer discrimination. Further, I find that an individual's exit probability increases leading up to benefit exhaustion, and remains constant after. This is consistent with search theory which would predict that individuals increase their search effort or lower their reservation wages until they reach benefit exhaustion. After that point, if there are no further changes in workers' incentives, their probability of exiting unemployment should remain constant. Interestingly, in their audit study, [Kroft et al. \(2013\)](#) find a decrease in callback rates only during the first six months of unemployment (refer to Figure 2 in their paper). In Section VI, I formally illustrate that my findings are consistent with a search model incorporating heterogeneous workers and falling callback rates early in the unemployment spell.

In recent studies, researchers have introduced behavioral modifications to the standard search theory in order to explain the observed decline in exit rates after UI exhaustion, which deviates from the predictions of the standard search model. Most notably, [DellaVigna et al. \(2021\)](#) introduces reference dependence in utility to account for this decline. However, after adjusting for compositional effects, I do not find evidence of a

decline in individual exit probabilities after UI exhaustion. Hence, I show that the data can be reconciled with the standard model by incorporating heterogeneous workers.

V.B Robustness

In this section, I examine the robustness of the main results by using different moments for estimation, considering alternative functional form restrictions on the baseline hazard, and relaxing assumptions of the baseline model.

Online Appendix E presents results from several robustness checks. Figures E.1 and E.2 demonstrate that estimates of the structural hazard remain largely unchanged when considering only two notice lengths (1-2 months or >2 months) or when using unweighted data. As shown in Section III, the structural hazard is non-parametrically identified. However, the standard errors for the non-parametric estimates are too large, rendering the estimates uninformative. Hence, I impose a log-logistic functional form on the hazard to minimize the number of estimated parameters. Nevertheless, I present the non-parametric estimates in Figure E.3. While the non-parametric hazard declines even after benefit extension, it doesn't fall below the hazard in the preceding interval of UI exhaustion, similar to the baseline estimate but in contrast to the observed hazard.

Figure E.4 displays estimates using data where unemployment duration is binned into 9-week intervals. The estimates qualitatively align with the baseline results. The estimated structural hazard is above the observed hazard at all durations, rises more than the observed hazard until UI exhaustion, and is constant after. Lastly, Figure E.5 provides separate estimates of the model for individuals who experienced displacement during years when UI benefits were potentially extended beyond 26 weeks. In this case, the structural hazard is slightly elevated relative to the baseline after benefit exhaustion.

In Online Appendix F, I relax the assumptions of my model in two dimensions. First, I allow the mean of the heterogeneity distribution to be different for workers with varying lengths of notice. Second, I let the structural hazards beyond the initial period vary

for workers with different lengths of notice up to some constant. I estimate the model for different parameter values and display the residuals for different values and the estimated structural hazard corresponding to the values that minimize the residuals in Figure F.3. The values that minimize the residuals suggest no mean differences between the two groups but imply a higher structural hazard for long-notice workers, even beyond the first 12 weeks. Consequently, the implied structural hazard from this exercise is higher than the baseline estimate. This discrepancy arises because the baseline estimate assumes no differences in the structural hazard between long and short-notice workers. However, if the structural hazard for long-notice workers is indeed higher, the composition-related gap in exit rates would be even greater than what was assumed in the baseline estimation.

VI A MODEL OF JOB-SEARCH

The estimates obtained from the Mixed Hazard model suggest a decline in an individual worker's probability of exiting unemployment during the initial five months. Additionally, I find evidence that an individual's likelihood of exiting unemployment increases as they approach the exhaustion of unemployment insurance (UI) benefits, and remains constant thereafter. The latter is in contrast to the observed exit rate, which continues to decline even after benefit exhaustion. Researchers have tried to explain the decline in the observed rate after exhaustion using behavioral explanations such as storable offers (Boone and van Ours, 2012) or reference dependence in utility (DellaVigna et al., 2021). In this section, I show that my findings align with standard search theory, incorporating heterogeneous workers, and are consistent with evidence from the audit study conducted by Kroft et al. (2013), which documents an initial decline in callback rates during the unemployment spell.

In particular, I set up a search model with heterogeneous workers. Within this model, workers choose search effort to maximize their expected utility. The likelihood of find-

ing a job depends on the offer arrival rate and a worker's search effort. Moreover, the offer arrival rate varies by the duration of unemployment and the type of worker. I calibrate the model to match the implied structural dependence to my estimate from the Mixed Hazard model and also match the exit rate implied by the model to the data. I then examine the trajectory of the offer arrival rate and search effort. This exercise also allows me to discern the impact on exit probabilities arising from two sources: the actions of optimizing agents in response to changing incentives and external factors directly influencing a worker's employment prospects.

VI.A Model Setup

I consider a stylized model of job search where a worker's search environment is non-stationary (Mortensen, 1986; Van Den Berg, 1990) and workers are heterogeneous. At every duration d , workers choose how much search effort s to exert to maximize their discounted expected utility.²⁸ Costs of search effort are given by the function $c(s)$, which is increasing, convex, and twice continuously differentiable, with $c(0) = 0$ and $c'(0) = 0$. The probability that a worker finds a job $\lambda(d, \nu, s)$ depends on the time elapsed since unemployed d , their search effort s , and their type ν as follows: $\lambda(d, \nu, s) = \delta(d)\nu s$. Here, $\delta(d)\nu$ is the offer arrival rate, which varies over the duration of unemployment and across workers of different types. Once workers find a job, they remain employed forever. A worker receives unemployment insurance (UI) benefits $b(d)$ when unemployed and wages w when employed. The function $u(\cdot)$ gives the flow utility from consumption. Then the value function for a worker of type ν unemployed at duration d is given by:

$$V_u(d, \nu) = \max_s u(b(d)) - c(s) + \beta [\lambda(s, d, \nu)V_e + (1 - \lambda(s, d, \nu))V_u(d + 1, \nu)]$$

²⁸Alternatively, the model could feature a reservation wage choice, and all conclusions about search effort would instead be regarding reservation wages.

Here, β is the discount rate, and V_e is the value of employment given by $V_e = u(w) + \beta V_e$. The UI benefits $b(d)$ are equal to b for $d \leq D_B$ and equal to 0 otherwise. I also assume that after some time $D_T \geq D_B$ the job-finding function $\lambda(d, s, \nu)$ does not depend on the duration of unemployment d , such that for $d > D_T$, $\delta(d) = \delta_T$. This ensures that after D_T , jobseekers face a stationary environment, and hence we can solve for the optimal search strategy of each worker in each period using backward induction. Finally, I consider the case of two types of workers: a high type H and a low type L with $\nu_H > \nu_L$, with π denoting the share of workers with the higher arrival rate.

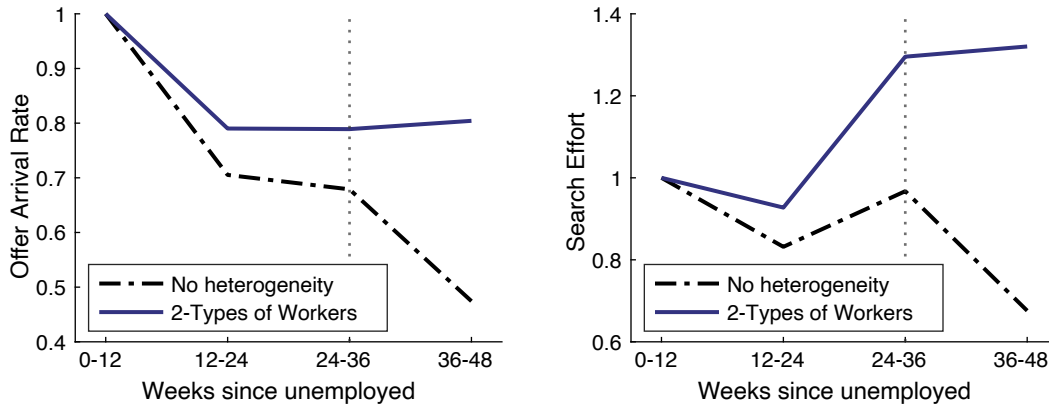
VI.B Numerical Analysis

I now calibrate the model specified in the previous section. Let $s(d, \nu)$ denote a worker's optimal search effort at duration d . The probability that this worker finds a job $h(d|\nu)$ is given by $\delta(d)s(d, \nu)\nu$. So a worker's exit rate evolves over the spell of unemployment due to changes in the offer arrival rate $\delta(d)$ and the worker's search effort. However, just as before, the observed exit rate $\tilde{h}(d) = \mathbb{E}[h(d|\nu)|D \geq d]$ also changes due to changes in composition over the spell of unemployment. I use my estimate of the structural hazard from the Mixed Hazard model to target structural duration dependence $h(d) = \mathbb{E}[h(d|\nu)]$ from the model.²⁹ Additionally, I match the exit rate implied by the model $\tilde{h}(d)$ to the data. In order to compare the predictions from this model to a model with no heterogeneity, I also calibrate the model assuming just one type of worker. In this case, I match the structural duration dependence or the exit rate implied by the model to the exit rate in the data. Further details for the calibration are provided in [Appendix B](#).

Table [B.2](#) shows that both the model with and without heterogeneity fit the data

²⁹Note that the search model does not correspond precisely to the econometric framework since it does not imply that $s(d, \nu)$ evolves in the same manner for each type of worker. However, in [Online Appendix G](#), I simulate data from the search model with notice periods and show that my estimator does reasonably well in capturing movements in $\mathbb{E}[h(d|\nu)]$.

FIGURE 3: CALIBRATION OF THE SEARCH MODEL



(A) OFFER ARRIVAL RATE

(B) SEARCH EFFORT

Notes: The figure presents the search effort and offer arrival rate from the calibration of the search model, assuming no heterogeneity (dashed line) and assuming two types of workers (solid line). The search effort is averaged over two types of workers.

perfectly. Figure 3 presents the search effort and offer arrival rate implied by the two calibration exercises. The offer arrival rate implied by the model with heterogeneity declines during the first five months and is constant after that, which is consistent with evidence from Kroft et al. (2013). Conversely, in the model with only one type of worker, the offer arrival rate continues to decline throughout the spell of unemployment. Finally, the model calibration implies that an individual’s search effort decreases slightly during the first five months but then increases up to UI exhaustion and remains stable after that. In summary, the data and my findings can be rationalized with conventional search theory, without any behavioral adjustments, but by incorporating heterogeneity among workers and declining returns to search early in the unemployment spell.

VII CONCLUSION

In this paper, I use a novel source of variation to disentangle the role of structural duration dependence from heterogeneity in the dynamics of the observed exit rate. I

document that workers who receive a longer notice before being laid off are more likely to exit unemployment early in the spell. However, the observed exit rate is lower for long-notice workers at later durations. This points towards the presence of heterogeneity across workers. As a higher proportion of the more employable workers from the long-notice group exit early, the composition of surviving long-notice workers at later durations is worse. I utilize these reduced-form moments and estimate a Mixed Hazard model.

The estimates from the hazard model uncover substantial heterogeneity in individual exit probabilities. The observed exit rate declines by about 41% over the first five months. In contrast, the estimated individual hazard only declines by 23% over this period. Moreover, I find that after the first five months, none of the depreciation in the observed exit rate is due to structural duration dependence. Instead, an individual's exit probability increases up to UI exhaustion and remains constant after. The observed exit rate continues to decline after exhaustion as well, which has led researchers to suggest behavioral explanations for this pattern. I provide an alternative explanation for this pattern which is the presence of heterogeneity. I show that my estimates can be rationalized within a standard search model with heterogeneous workers. These findings underscore the importance of incorporating heterogeneity when estimating and calibrating search models.

APPENDIX A PROOFS AND DERIVATIONS

This section presents the proofs of the results in the main text. Before we proceed, let's define the survival function $S(d) = 1 - G(d)$ as the probability of unemployment duration D being greater than d . Then we can write, $S(d) = \prod_{s=1}^d (1 - h(s))$. In which case, under Assumption 1, $h(d|v, l, X) = \psi_l(d, X)v$, we can write the conditional dura-

tion distribution as follows:

$$\begin{aligned} g(d|l, X) &= \mathbb{E}[S(d-1|\nu, l, X) - S(d|\nu, l, X)|l, X] \\ &= \psi_l(d, X) \mathbb{E}[\nu S(d-1|\nu, l, X)|l, X] \end{aligned} \quad (2)$$

Note that here $S(0|\nu, l, X) = 1$.

A.1 Proof of Proposition 1

Proof. Given equation (2), we can write:

$$1 - G(d-1|l, X) = \mathbb{E}[S(d-1|\nu, l, X)] = S(d-1|l, X)$$

In which case, we can write the observed exit rate as follows:

$$\tilde{h}(d|l, X) = \frac{g(d|l, X)}{1 - G(d-1|l, X)} = \psi_l(d, X) \mathbb{E} \left(\nu \cdot \frac{S(d-1|\nu, l, X)}{S(d-1|l, X)} \middle| l, X \right)$$

To see that the second term in the above expression is the average type $\mathbb{E}(\nu|D \geq d, l, X)$ amongst surviving workers at the beginning of d , note that

$$f(\nu|D \geq d, l, X) = \frac{\Pr(D > d-1|\nu, l, X)f(\nu|l, X)}{\Pr(D > d-1|l, X)} = \frac{S(d-1|\nu, l, X)f(\nu|l, X)}{S(d-1|l, X)}$$

where the first inequality follows from the Bayes rule.

Now, for any d and $\nu_H > \nu_L$,

$$\frac{S(d|\nu_H, l, X)}{S(d-1|\nu_H, l, X)} < \frac{S(d|\nu_L, l, X)}{S(d-1|\nu_L, l, X)}$$

The above equation implies that,

$$\frac{f(\nu_H|D \geq d+1, l, X)}{f(\nu_L|D \geq d+1, l, X)} < \frac{f(\nu_H|D \geq d, l, X)}{f(\nu_L|D \geq d, l, X)}$$

In which case, $f(\nu|D \geq d, l, X)$ first-order stochastically dominates $f(\nu|D \geq d+1, l, X)$ which implies that $\mathbb{E}(\nu|D \geq d, l, X) \geq \mathbb{E}(\nu|D \geq d+1, l, X)$. \square

A.2 Statement and Proof of Lemma 1

The following lemma states that the identification of structural hazards implies the identification of higher moments of the unobserved type distribution.

Lemma 1. *Under Assumption 1, if $\psi_l(d, X)$ is known for $d = 1, \dots, \bar{D}$, then we can identify the first \bar{D} conditional moments of v , given by $\{\mathbb{E}(v^k|l, X)\}_{k=1}^{\bar{D}}$, from the conditional unemployment $g(d|l, X)$ distribution for $d = 1, \dots, \bar{D}$.*

Proof. Expanding equation (2) for $d = 1, 2, 3, \dots$, we can write:

$$\begin{aligned} g(1|l, X) &= \psi_l(1, X)\mathbb{E}(v|l, X) \\ g(2|l, X) &= \psi_l(2, X)[\mathbb{E}(v|l, X) - \psi_l(1, X)\mathbb{E}(v^2|l, X)] \\ g(3|l, X) &= \psi_l(3, X)[\mathbb{E}(v|l, X) - [\psi_l(1, X) + \psi_l(2, X)]\mathbb{E}(v^2|l, X) + \psi_l(1, X)\psi_l(2, X)\mathbb{E}(v^3|l, X)] \\ &\vdots \end{aligned}$$

Or, more compactly,

$$g(d|l, X) = \psi_l(d, X) \sum_{k=1}^{\bar{D}} c_k(d, \boldsymbol{\psi}_{l, X}) \mathbb{E}(v^k|l, X) \quad (3)$$

where $\boldsymbol{\psi}_{l, X} = \{\psi_l(d, X)\}_{d=1}^{\bar{D}}$ and

$$c_k(d, \boldsymbol{\psi}_{l, X}) = \begin{cases} 1 & \text{for } k = 1 \\ c_k(d-1, \boldsymbol{\psi}_{l, X}) - \psi_l(d-1, X)c_{k-1}(d-1, \boldsymbol{\psi}_{l, X}) & \text{for } 1 \leq k \leq d \\ 0 & \text{for } k > d \end{cases}$$

Denote $\mathbf{g}_{l, X} = \{g(d|l, X)\}_{d=1}^{\bar{D}}$ and $\boldsymbol{\mu}_{l, X} = \{\mathbb{E}(v^k|l, X)\}_{k=1}^{\bar{D}}$. Then we can write $\mathbf{g}_{l, X} = C(\boldsymbol{\psi}_{l, X})\boldsymbol{\mu}_{l, X}$ where $C(\boldsymbol{\psi}_{l, X})$ is the $\bar{D} \times \bar{D}$ upper triangular matrix with $C_{s, k}(\boldsymbol{\psi}_{l, X}) = \psi_l(s, X)c_k(s, \boldsymbol{\psi}_{l, X})$. In addition, the diagonal elements of $C(\boldsymbol{\psi}_{l, X})$ are non-zero. To see this note that, $C_{d, d}(\boldsymbol{\psi}_{l, X}) = (-1)^{d-1} \prod_{s=1}^d \psi_l(s, X)$ and each $\psi_l(s, X) > 0$. Hence,

$C(\psi_{l,X})$ is invertible and we can plug in $\psi_{l,X}$ in $\mathbf{g}_{l,X} = C(\psi_{l,X})\boldsymbol{\mu}_{l,X}$ to solve for $\boldsymbol{\mu}_{l,X}$. \square

A.3 Proof of Theorem 1

Proof. Define $\tilde{S}(d|\nu, X)$ as follows,

$$\tilde{S}(d|\nu, X) = \prod_{s=2}^d [1 - \psi(s, X)\nu]$$

By Assumption 3, we know that $\psi_l(d, X) = \psi(d, X)$ for $d > 1$. Therefore, for all l and $d > 1$, we have:

$$S(d|\nu, l, X) = [1 - \psi_l(1, X)\nu]\tilde{S}(d|\nu, X) \quad (4)$$

Note that $1 - G(d|l, X) = \mathbb{E}[S(d|\nu, l, X)|l, X]$. By Assumption 2, we have $\nu \perp L|X$. Therefore, we can write:

$$1 - G(d|l, X) = \mathbb{E}[S(d|\nu, l, X)|X] \quad (5)$$

In which case,

$$g(d|l, X) = \psi_l(d, X)\mathbb{E}[\nu S(d-1|\nu, l, X)|X] \quad (6)$$

Now consider any l, l' , plugging in equation (4) in equations (5) and (6) and taking the difference between l' and l , we get

$$G(d-1|l', X) - G(d-1|l, X) = [\psi_{l'}(1, X) - \psi_l(1, X)]\mathbb{E}[\nu\tilde{S}(d-1|\nu, X)|X] \quad (7)$$

$$g(d|l', X) - g(d|l, X) = -\psi(d, X)[\psi_{l'}(1, X) - \psi_l(1, X)]\mathbb{E}[\nu^2\tilde{S}(d-1|\nu, X)|X] \quad (8)$$

From equation (7), we can write:

$$\mathbb{E}[\nu\tilde{S}(d-1|\nu, X)|X] = \frac{G(d-1|l', X) - G(d-1|l, X)}{\psi_{l'}(1, X) - \psi_l(1, X)} \quad (9)$$

Similarly, from equation (8), we have:

$$\mathbb{E}[\nu^2 \tilde{S}(d-1|\nu, X)|X] = -\frac{g(d|l', X) - g(d|l, X)}{\psi(d, X)(\psi_{l'}(1, X) - \psi_l(1, X))} \quad (10)$$

Note that plugging in the expression for $S(d-1|\nu, l, X)$ from equation (4) in equation (6) gives us,

$$g(d|l, X) = \psi(d, X) \left[\mathbb{E}[\nu \tilde{S}(d-1|\nu, X)|X] - \psi_l(1, X) \mathbb{E}[\nu^2 \tilde{S}(d-1|\nu, X)|X] \right]$$

Plugging the expressions from equations (9) and (10) in the above equation, for $d > 1$ we can find:

$$\psi(d, X) = \frac{g(d|l, X)\psi_{l'}(1, X) - g(d|l', X)\psi_l(1, X)}{G(d-1|l', X) - G(d-1|l, X)}$$

Here, the denominator is not equal to zero as we assumed $\psi_{l'}'(1, X) \neq \psi_l'(1, X)$.

Now note that for $d = 1$, $g(1|l, X) = \psi_l(1, X)\mathbb{E}[\nu|X]$. So plugging in $\psi_l(1, X) = g(1|l, X)/\mathbb{E}[\nu|X]$ in the expression for $\psi(d, X)$ above, we can write:

$$\psi(d, X)\mathbb{E}(\nu|X) = \frac{g(d|l, X)g(1|l', X) - g(d|l', X)g(1|l, X)}{G(d-1|l', X) - G(d-1|l, X)} \quad (11)$$

This proves the identification of $\{\psi_l(1, X), \psi_{l'}(1, X), \{\psi(d, X)\}_{d=2}^D\}$ up to a scale. Identification of moments follows from Lemma 1. \square

A.4 Proof of Corollary 1

Proof. Note that, $1 - G(d|l) = \mathbb{E}[S(d|\nu, l)|l]$. Then under the assumption $L \perp \nu$, we can write:

$$1 - G(d|l) = \mathbb{E}[S(d|\nu, l)]$$

In which case,

$$g(d|l) = \psi_l(d)\mathbb{E}[\nu S(d-1|\nu, l)]$$

Following similar steps as in the proof for Theorem 1, we can write for $d > 1$,

$$\psi(d)\mathbb{E}(v) = \frac{g(d|l)g(1|l') - g(d|l')g(1|l)}{G(d-1|l') - G(d-1|l)}$$

Identification of moments follows from Lemma 1. □

A.5 Proof of Proposition 2

Proof. Plugging in $\psi(d, X) = \psi(d)\phi(X)$ in equation (11) and rearranging, we can write:

$$\psi(d)\phi(X)\mathbb{E}(v|X)[G(d-1|l', X) - G(d-1|l, X)] = g(d|l, X)g(1|l', X) - g(d|l', X)g(1|l, X)$$

If we integrate the above expression using the weighted distribution of X , denoted by $F_X^\omega(\cdot)$, then the following expression holds for $d > 1$:

$$\psi(d) \left[\int_{\mathcal{X}} \phi(x)\mathbb{E}(v|x)\partial F_X^\omega(x) \right] = \frac{G^\omega(d|l)G^\omega(1|l') - G^\omega(d|l')G^\omega(1|l)}{G^\omega(d-1|l') - G^\omega(d-1|l)}$$

$G^\omega(d|l)$ denotes the weighted unemployment duration distribution. Note that first-period hazards can be recovered from $G^\omega(1|l')$ and $G^\omega(1|l)$. This proves the identification of structural hazards $\{\psi_l(1), \psi_{l'}(1), \{\psi(d)\}_{d=2}^{\bar{D}}\}$ up to a scale.

To see that the adjusted moments are identified as well, note that since $\psi_l(d, X) = \psi_l(d)\phi(X)$ and $f(v|l, X) = f(v|X)$, we can write equation (3) as:

$$g(d|l, X) = \psi_l(d) \sum_{k=1}^{\bar{D}} c_k(d, \psi_l) \phi(X)^k \mathbb{E}(v^k|X)$$

where $\boldsymbol{\psi}_l = \{\psi_l(d)\}_{d=1}^{\bar{D}}$ and

$$c_k(d, \boldsymbol{\psi}_l) = \begin{cases} 1 & \text{for } k = 1 \\ c_k(d-1, \boldsymbol{\psi}_l) - \psi_l(d-1)c_{k-1}(d-1, \boldsymbol{\psi}_l) & \text{for } 1 \leq k \leq d \\ 0 & \text{for } k > d \end{cases}$$

Integrating the above expression for $g(d|l, X)$ using the weighted distribution of X , we can write:

$$G^\omega(d|l) = \psi_l(d) \sum_{k=1}^{\bar{D}} c_k(d, \boldsymbol{\psi}_l) \mu_k^\omega$$

where $\mu_k^\omega = \int_{\mathcal{X}} \phi(x)^k \mathbb{E}(\nu^k | x) \partial F_X^\omega(x)$. The rest of the proof follows as in the proof for Lemma 1. Denote $\mathbf{G}_l^\omega = \{G^\omega(d|l)\}_{d=1}^{\bar{D}}$ and $\boldsymbol{\mu}^\omega = \{\mu_k^\omega\}_{k=1}^{\bar{D}}$. Then we can write $\mathbf{G}_l^\omega = C(\boldsymbol{\psi}_l) \boldsymbol{\mu}^\omega$ where $C(\boldsymbol{\psi}_l)$ is the $\bar{D} \times \bar{D}$ upper triangular matrix with $C_{s,k}(\boldsymbol{\psi}_l) = \psi_l(s)c_k(s, \boldsymbol{\psi}_l)$. In addition, the diagonal elements of $C(\boldsymbol{\psi}_l)$ are non-zero as $C_{d,d}(\boldsymbol{\psi}_l) = (-1)^{d-1} \prod_{s=1}^d \psi_l(s)$ and each $\psi_l(s) > 0$. Hence, $C(\boldsymbol{\psi}_l)$ is invertible and we can plug in $\boldsymbol{\psi}_l$ in $\mathbf{g}_l = C(\boldsymbol{\psi}_l) \boldsymbol{\mu}_l^\omega$ to solve for $\boldsymbol{\mu}_l^\omega$.

□

A.6 Proof that $\mathbb{E}[m_i(l, d, \Theta)] = 0$

In this subsection, I demonstrate that the expected value of the moment condition, $\mathbb{E}[m_i(l, d, \Theta)]$, is equal to zero.

Proof. Note that,

$$m(l, d, \Theta) = \mathbb{I}\{L = l\} w [\mathbb{I}\{D = d\} - g^\omega(d|l; \Theta)]$$

where $g^\omega(d|l; \Theta) = \int g(d|l, x)w_l(x)f_X(x|l)\partial x$. Now, note that,

$$\begin{aligned}
\mathbb{E}[\mathbb{I}\{L_i = l\}w_i\mathbb{I}\{D_i = d\}] &= \mathbb{E}[\mathbb{E}[\mathbb{I}\{L_i = l\}w_i\mathbb{I}\{D_i = d\}|X_i = x]] \\
&= \mathbb{E}[\mathbb{E}[w_i\mathbb{I}\{D_i = d\}|X_i = x, L_i = l]Pr(L_i = 1|X_i = x)] \\
&= \mathbb{E}[w_l(x)\mathbb{E}[\mathbb{I}\{D_i = d\}|X_i = x, L_i = l]Pr(L_i = 1|X_i = x)] \\
&= \mathbb{E}[w_l(x)g(d|l, x)Pr(L_i = l|X_i = x)] \\
&= \int w_l(x)g(d|l, x)Pr(L_i = l|X_i = x)f_X(x)\partial x
\end{aligned}$$

Let $p_l = Pr(L = l)$ and note that $Pr(L_i = l|X_i = x)f_X(x) = f_X(x|l)p_l$. Then we can write, $\mathbb{E}[\mathbb{I}\{L_i = l\}w_i\mathbb{I}\{D_i = d\}] = p_l g^\omega(d|l; \Theta)$. Finally, note that $\mathbb{E}[\mathbb{I}\{L = l\}g^\omega(d|l; \Theta)] = p_l g^\omega(d|l; \Theta)$ which implies that $\mathbb{E}[m(l, d, \Theta)] = 0$. \square

APPENDIX B SEARCH MODEL CALIBRATION

I calibrate the model under standard values for model parameters. To maintain consistency with the econometric model, each period is assumed to be 12 weeks long. Corresponding to a 5 percent annual interest rate, the discount factor β is set equal to 0.985. I normalize the wage to 1 and set the replacement rate for unemployment benefits at 0.5. In addition, I assume individuals receive an annuity payment of 0.1 times their wages in each period, regardless of their employment status. This can be interpreted as the income of a secondary earner. Utility from consumption is given by the constant relative risk aversion (CRRA) utility function, $u(c) = c^{1-\sigma}/(1-\sigma)$ with $\sigma = 1.75$. I follow [DellaVigna et al. \(2017\)](#) and [Marinescu and Skandalis \(2021\)](#), assume that costs of job search are given by $c(s) = \theta s^{1+\rho}/(1+\rho)$. I set $\rho = 1$ and $\theta = 50$.

³⁰ Table [B.1](#) summarizes the calibration parameters. Table [B.2](#) displays the fit of the

³⁰Different parameters for the cost function do not change qualitative predictions of my exercise but do lead to changes in the scale of the search effort.

calibrated model.

TABLE B.1: CALIBRATION PARAMETERS FOR THE SEARCH MODEL

Parameter	Value
Length of each period	12 Weeks
Discount factor β	0.985
Relative risk aversion σ	1.75
Per period wages w	1
Annuity Payments	0.1
Unemployment benefits	0.5
Benefit exhaustion D_B	3
Search cost parameter ρ	1
Search cost parameter θ	50
First period arrival rate $\delta(1)$	1

Note: The table presents the parameters used for calibrating the search model in Section VI.

TABLE B.2: SEARCH MODEL CALIBRATION: FIT

D	Observed Hazard			Structural Hazard	
	Data (1)	Model with 1 type (2)	Model with 2 types (3)	MH Estimate (4)	Model with 2 types (5)
1	0.521	0.521	0.521	0.521	0.521
2	0.305	0.305	0.305	0.399	0.399
3	0.342	0.342	0.342	0.607	0.607
4	0.167	0.167	0.167	0.630	0.630

Note: The table displays the exit rate from the data in column (1) and the corresponding fitted values while calibrating the search model with one type of worker in column (2) and two types of workers in column (3). Columns (4) and (5) present the estimated structural hazard from the Mixed Hazard (MH) model and the fitted structural hazard when calibrating the search model with two types of workers.

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