Online Appendix

Duration Dependence and Heterogeneity: Learning from Early Notice of Layoff

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Appendix C Data

C.1 Data Construction and Sample Selection

The Displaced Worker Supplement (DWS) was introduced in 1984, but the variable on the length of notice was not included in the first two samples. Furthermore, the definition of displaced workers has undergone changes over time.[1] Before 1998, self-employed individuals or those who expected to be recalled to their lost job within six months were also included in the survey. However, the information on whether a worker expected to be recalled is only available for the years 1994 and 1996. In addition, the data on the length of time individuals took to find their next job is miscoded and largely missing for the year 1994. For these reasons, my analysis begins from 1996. Moreover, to maintain consistency across years, I exclude self-employed individuals or those who expected to be recalled from the 1996 sample.

The duration of unemployment for individuals who have secured a job by the time of the survey is given by the *dwwksun* variable, which measures the number of weeks the person was unemployed between leaving or losing one job and starting another. For those who report not holding another job since their last job, censored duration is obtained using the *durunemp* variable from the CPS. Only individuals with non-missing

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[1] The recall window was 5 years instead of 3 before 1994.

Table C.1: Comparison of the analytical sample to all individuals in the Displaced Worker Supplement (DWS) and the Current Population Survey (CPS)

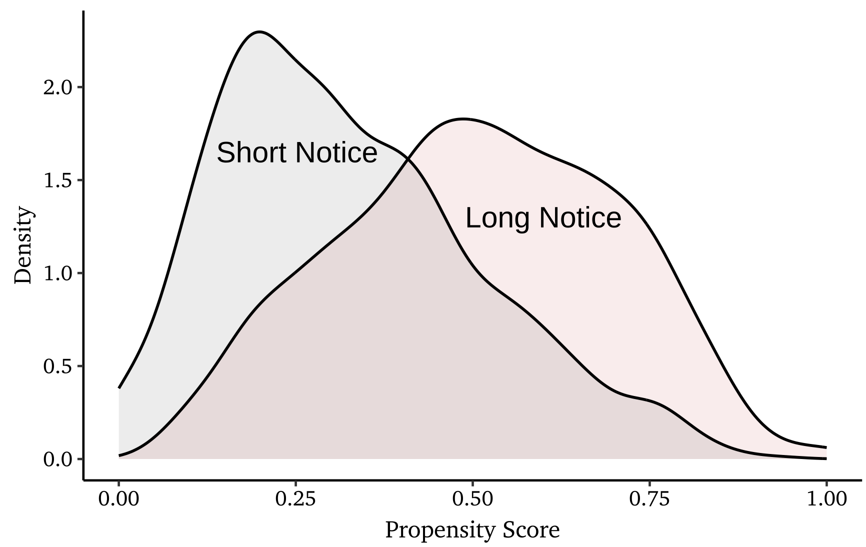
	Sample	DWS	CPS
	(1)	(2)	(3)
Age	42.87	40.61	42.17
Female	0.44	0.44	0.52
Black	0.09	0.11	0.10
Married	0.61	0.54	0.60
Educational Attainment			
HS Dropout	0.04	0.09	0.09
HS Graduate	0.57	0.65	0.60
College Degree	0.39	0.26	0.30
Employment Status			
Employed	0.89	0.67	0.74
Unemployed	0.09	0.21	0.04
NILF	0.02	0.12	0.21
Observations	3556	44707	969604

Note: All samples are restricted to individuals between the ages of 21 to 64 and pertain to years 1996-2020. Column (1) includes individuals from the DWS who lost their job at least one year before the survey, worked full-time for at least six months and were provided health insurance at their lost job, did not expect to be recalled, and received a layoff notice. Columns (2) and (3) include all individuals in the DWS and the monthly CPS, respectively, over the sample period.

information on their unemployment duration are included in my sample. In addition, since the sample is restricted to individuals who lost a job at least one year prior to the survey, any individuals who haven't found a job but report an unemployment duration of less than a year are excluded from the sample. Moreover, individuals with missing information on earnings, industry, or occupation at the previous job are also excluded from the sample. Finally, to minimize retrospective bias, I exclude individuals who report switching more than two jobs since losing their previous job.

Since 2012, tenure at the lost job was top-coded at 24 years. To maintain consistency across samples, I also implement a top code of 24 years for all years prior to 2012. Earnings are reported in 1999 dollars. Table C.1 presents the descriptive statistics of my analytical sample compared to all individuals in the DWS as well as the CPS over the sample period. Relative to the CPS and DWS, individuals in the sample are more

FIGURE C.1: ASSESSING OVERLAP OF PROPENSITY SCORE DISTRIBUTIONS



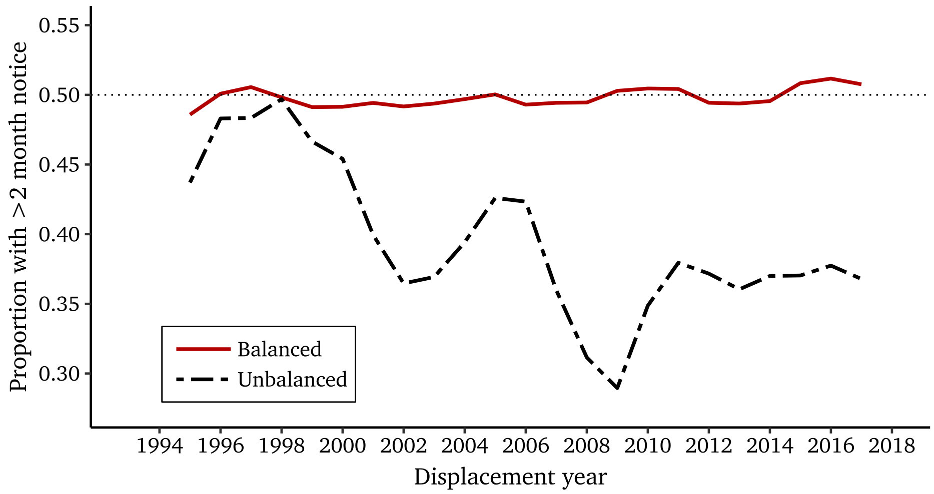
Note: The figure presents the density of estimated propensity scores for individuals with short and long notice separately.

educated and have higher employment rates.

C.2 Propensity Score Weighting

To ensure individuals with long and short notice are comparable, I reweight the sample using inverse propensity score weighting. The weight for each individual is calculated as the inverse of the likelihood of receiving the reported notice length. To estimate the propensity scores, I utilize a logistic regression where the odds of receiving a longer notice are modeled as a function of several variables. These variables include age, gender, marital status, race (indicator for Black), college education, being laid off due to plant closure, membership in a union, residing in a metropolitan area, tenure and earnings at the lost job, occupation at the lost job, state fixed effects, and the interaction between displacement year and industry of the lost job fixed effects. The density of estimated propensity scores for short and long-notice individuals is displayed in Figure C.1. The figure shows that there is a significant overlap between the two distributions,

FIGURE C.2: LENGTH OF NOTICE OVER TIME



Note: The figure plots a 3-year moving average of the proportion of individuals who received a notice of more than 2 months amongst all individuals in the sample who were displaced in a given year.

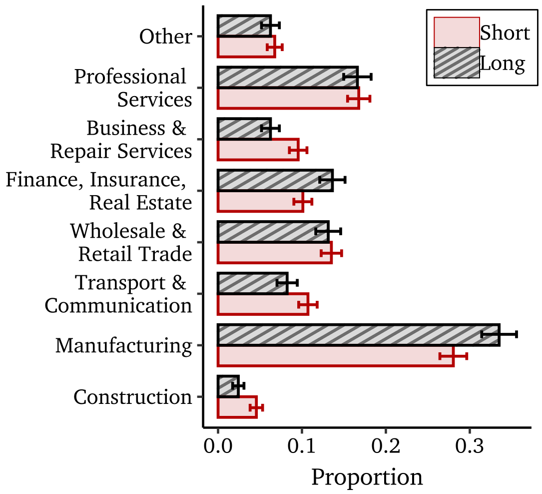
making further trimming of the data unnecessary.

Table 1 in the main text provides evidence that the reweighting achieves balance across certain observable variables. Figure C.2 demonstrates that reweighting leads to balance with respect to the year of displacement. In addition, Figure C.3 presents occupation and industry distributions for short and long-notice workers in both the balanced and unbalanced samples. Notably, the weighted sample exhibits more similarity in the industrial and occupational composition of short and long-notice workers.

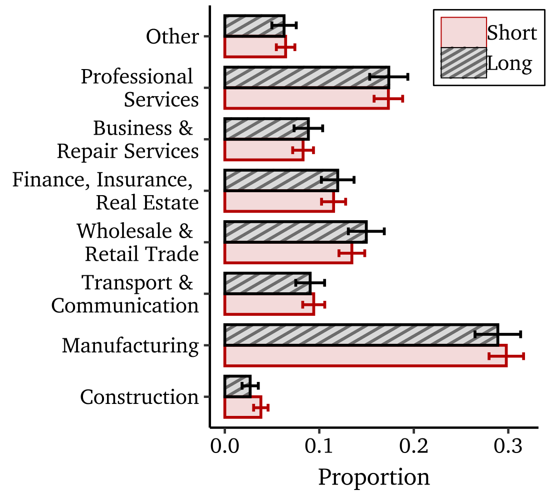
C.3 Additional Descriptives

This section provides additional descriptive statistics. Table C.2 presents the relationship between longer notice and earnings at the subsequent job. The table indicates that workers with longer notice tend to have higher earnings in their subsequent jobs. However, we cannot interpret this as a direct impact of longer notice because extended periods of unemployment can have a negative impact on wages (**?**), and as shown in this paper, a longer notice leads to shorter unemployment spells.

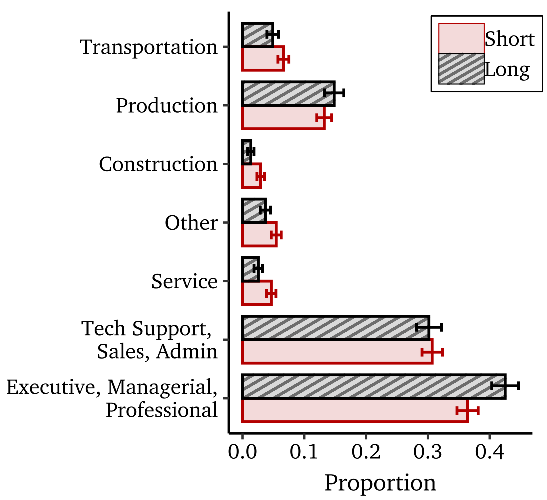
FIGURE C.3: INDUSTRY AND OCCUPATION OF THE LOST JOB



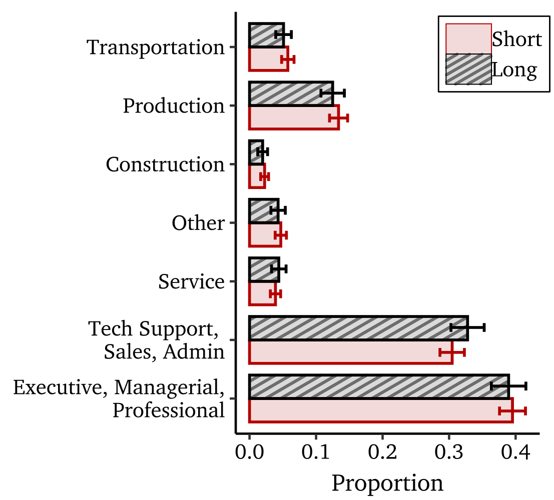
(A) INDUSTRY, UNBALANCED



(B) INDUSTRY, BALANCED



(C) OCCUPATION, UNBALANCED



(D) OCCUPATION, BALANCED

Note: The figure presents the proportions of individuals whose displaced jobs were in specific industries (panels A and B) and occupations (panels C and D) among long-notice and short-notice workers in both the unbalanced and balanced samples. The error bars represent the 90% confidence intervals.

Table C.2: Earnings at the Subsequent Job

	Weekly Log Earnings			
	(1)	(2)	(3)	(4)
>2 month notice	0.144***	0.129***	0.130***	0.126***
	(0.041)	(0.036)	(0.044)	(0.034)
Controls	No	Yes	No	Yes
Weights	No	No	Yes	Yes
	2370	2370	2370	2370

Note: The table shows results from linear regressions of log weekly wages at the subsequent job on an indicator for receiving a notice of more than 2 months. The sample used is similar to the main analytical sample, but it excludes individuals who had not yet found employment at the time of the survey, had multiple jobs between their previous and current job, or had incomplete earnings information for other reasons. Robust standard errors are reported in the parenthesis.

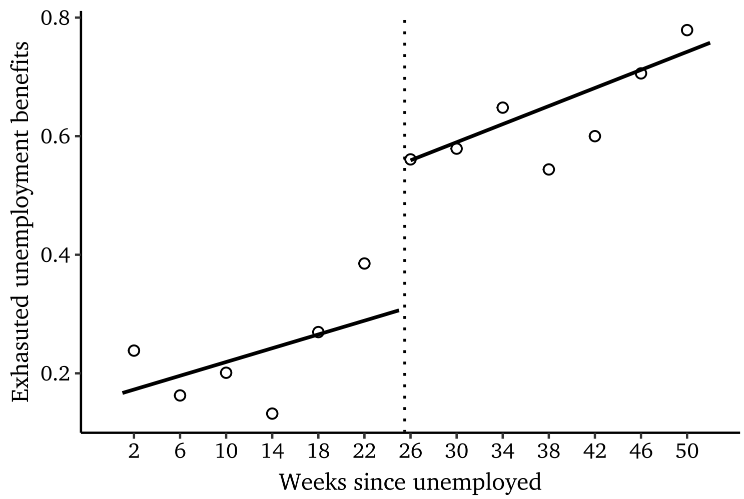
Table C.3 describes the incidence of UI take-up in the sample. Figures C.4 and C.5 describe the timing of benefit exhaustion amongst UI takers. Figure C.6 presents the data with unemployment duration binned in 4 and 9-week intervals. Figure C.7 displays the fitted hazard from the Cox Mixed Proportional hazard model after accounting for a comprehensive set of observable characteristics. Finally, Figure C.8 presents the distribution of notice length from the Survey of Consumer Expectations (SCE).

Table C.3: Unemployment Insurance Take-up

Unemployment Duration	Observations	Recieved UI Benefits
0 Weeks	591	0.07
0-4 Weeks	797	0.30
4-8 Weeks	335	0.63
8-12 Weeks	303	0.69
>12 Weeks	1516	0.83

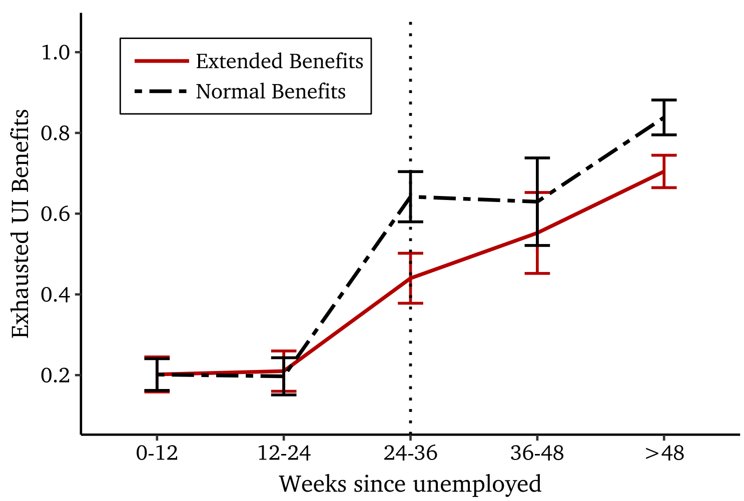
Notes: This table reports the percentage of individuals in the baseline sample who reported receiving UI benefits by the duration of unemployment.

Figure C.4: Timing of Benefit Exhaustion



Note: The figure presents the proportion of individuals who report having exhausted their UI benefits by the duration of unemployment. The sample is restricted to individuals in the main analytical sample who reported receiving UI benefits, and duration is binned in 4-week intervals.

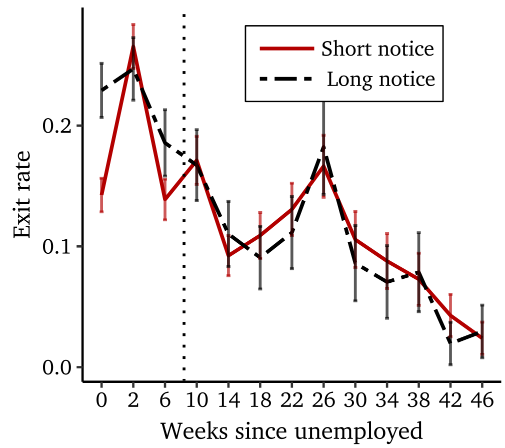
Figure C.5: Extended Benefit Years vs. Other Years



Note: The figure presents the proportion of individuals who report having exhausted their UI benefits by the duration of unemployment. The sample is restricted to individuals in the main analytical sample who reported receiving UI benefits. The solid line presents the proportion for those displaced during 2001-2004 or 2008-2013. While the dashed line presents the proportion for those displaced during other years in the sample.

FIGURE C.6: SURVIVAL AND EXIT RATES WITH ALTERNATIVE BINS

(A) SURVIVAL RATE



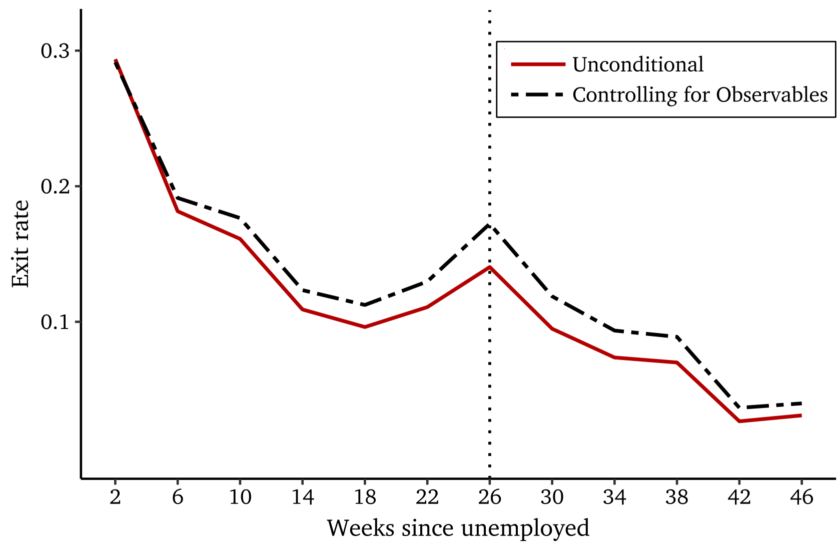
(B) EXIT RATE

(C) SURVIVAL RATE

(D) EXIT RATE

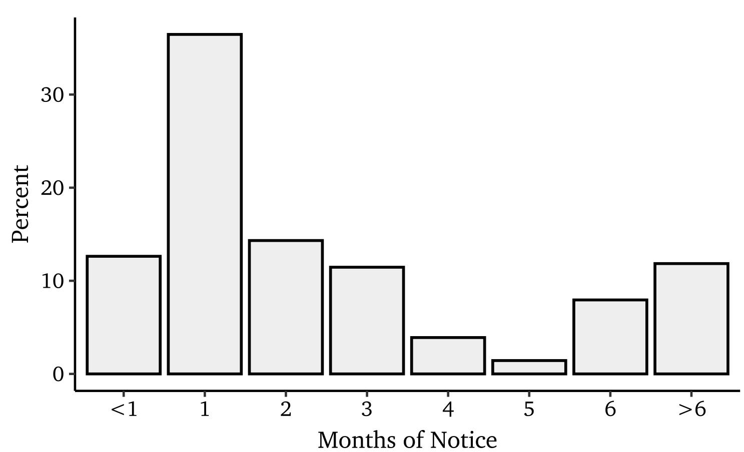
Note: Unemployment duration is binned in 4-week intervals for panels A and B, while it is binned in 9-week intervals for panels C and D. Panel A and C present the proportion of individuals who are unemployed at the beginning of each interval. Panel B and D present the proportion of individuals exiting unemployment in each interval amongst those who were still unemployed at the beginning of the interval. Error bars represent 90% confidence intervals.

Figure C.7: Cox Proportional Hazard Model



Note: The figure presents estimates of the structural hazard from the Cox proportional hazard model (`coxph` in R). The sample consists of 30,731 individuals from the DWS for the years 1996-2020 who worked full-time at their previous employer and did not expect to be recalled. Observations with missing values on unemployment duration are excluded. Observable characteristics controlled for include age, gender, race, education, marital status, the reason for displacement, union status, years of tenure and earnings at their last job, year of displacement, occupation, industry, and state of residence.

Figure C.8: Notice Length from SCE



Note: Data is from the Survey of Consumer Expectations (SCE) for the years 2013-2019. The sample consists of 768 individuals who received a layoff notice and reported the notice length.

Appendix D Additional Proofs

D.1 Proof of an Auxiliary Lemma

Lemma D.1. If $X \perp W|Z$ and $Y \perp W|Z$ for four random variables W, X, Y, and Z, then $f_{X|Y,Z,W}(x|y,z,w) = f_{X|Y,Z}(x|y,z)$.

Proof. Note that by the definition of conditional independence, we can write:

$$f_{X|Y,Z,W}(x|y,z,w) = \frac{f_{X,Y,Z,W}(x,y,z,w)}{f_{Y,Z,W}(y,z,w)} \tag{1}$$

Furthermore, the numerator in the above expression can be written as:

$$f_{X,Y,Z,W}(x,y,z,w) = f_{X,Y|Z,W}(x,y|z,w)f_{Z,W}(z,w) = f_{X,Y|Z}(x,y|z)f_{Z,W}(z,w)$$

The second equality in the above equation follows from $X \perp W|Z$ and $Y \perp W|Z$. Similarly, the denominator in equation (1) can be written as:

$$f_{Y,Z,W}(y,z,w) = f_{Y|Z,W}(y|z,w)f_{Z,W}(z,w) = f_{Y|Z}(y|z)f_{Z,W}(z,w)$$

Here, the second equality follows from $Y \perp W|Z$.
Plugging back the expressions for the numerator and denominator back into equation (1), we get:

$$f_{X|Y,Z,W}(x|y,z,w) = \frac{f_{X,Y|Z}(x,y|z)}{f_{Y|Z}(y|z)} = f_{X|Y,Z}(x|y,z)$$

□

D.2 Dealing with Censored Data

The identification result in the main text pertains to the distribution of completed unemployment durations. However, in many datasets, some individuals are still unemployed at the time of the survey. For these unemployed individuals, we observe how long they have been unemployed, but we do not know if and when they will find a job. Let D_C denote the censoring time, that is, the time elapsed since an individual becomes unemployed to the time of the survey. For individuals who have already exited unemployment at the time of the survey, we observe their completed unemployment duration D in the data. However, we only observe the censoring time D_C for currently unemployed individuals. Specifically, for each individual, we observe $\Delta = \min\{D, D_C\}$ along with an indicator variable for whether the individual was censored or not. Let $G_\Delta(.)$ denote the cumulative distribution of observed durations Δ.

The following result demonstrates that we can identify the structural hazard up to $\bar{D}$ if we assume that the censoring time D_C is independent of notice length conditional on observables.[2] We can achieve this by restricting our sample to individuals who were censored after $\bar{D}$. To understand why, note that we know the unemployment duration for individuals censored after $\bar{D}$ and report a duration of less than $\bar{D}$. Specifically, for any duration $d < \bar{D}$, we have $G^\Delta(d|l, X, D_C > \bar{D}) = G(d|l, X, D_C > \bar{D})$.[3]

Corollary D.1. Under Assumptions 1-3 and $D_C \perp L|X$, for any l, l' and some integer $\bar{D}$, the structural hazards $\{\psi_l(1, X), \psi_{l'}(1, X), \{\psi(d, X)\}_{d=2}^{\bar{D}}\}$ and the conditional moments of the type distribution $\{\mathbb{E}(\nu^d|X, D_C > \bar{D})_k\}_{d=1}^{\bar{D}}$ are identified up to scale from $\{G_\Delta(d|l, X, D_C > \bar{D}), G_\Delta(d|l', X, D_C > \bar{D}, l)\}_{d=1}^{\bar{D}}$.

Proof. First note that for $d < \bar{D}$,

$$
\begin{aligned}
G_\Delta(d|\nu, L, X, D_C > \bar{D}) &= 1 - Pr(\Delta > d|\nu, L, X, D_C > \bar{D}) \\
&= 1 - Pr(D > d, D_C > d|\nu, L, X, D_C > \bar{D}) \\
&= G(d|\nu, L, X, D_C > \bar{D})
\end{aligned}
$$

[2]It is common in the literature to assume that D_C is independent of ν, which would result in identification in the current model as well. However, this assumption is stronger than necessary in this specific context.

[3]In theory, it may be possible and more efficient to condition on $D_C > d$ at every duration d. However, in the DWS data, D_C is observed only at one-year intervals.

The second equality is due to $\Delta = \min\{D, D_C\}$ and the third equality follows from $d < \bar{D} < D_C$.

Given that $\nu \perp L|X$ and $D_C \perp L|X$, it follows that $f(\nu|L,X,D_C) = f(\nu|X,D_C)$. Online D.1 presents this statement and its proof. In which case, we can write

$$1 - G_{\Delta}(d|l,X,D_C > \bar{D}) = \mathbb{E}[S(d|\nu,l,X)|X,D_C > \bar{D}]$$

We can complete the proof by following the same steps as in the proof for Theorem 1, but by replacing moments conditional on L and X with moments conditional on L, X, and $D_C > \bar{D}$. □

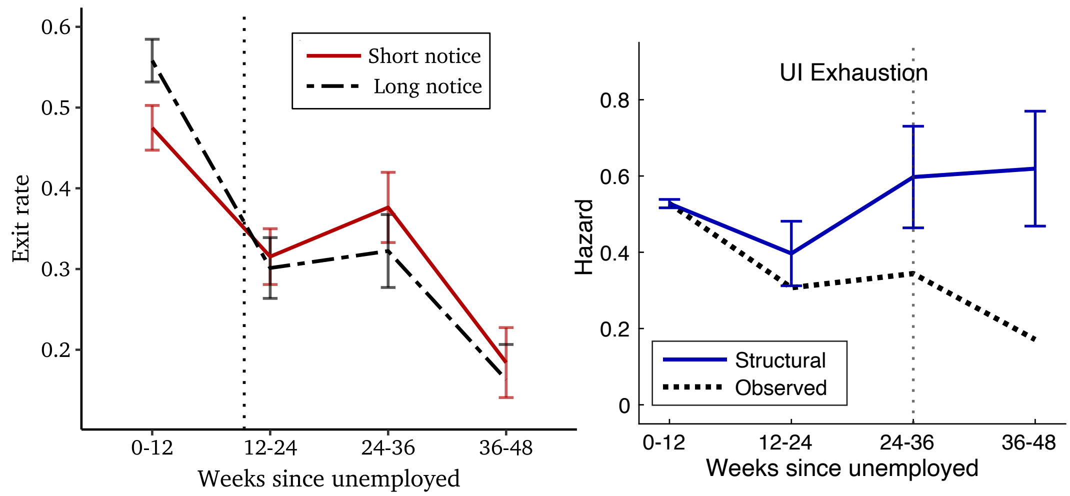
Based on Proposition 1 and the result above, we can deduce that if $h(d|\nu,l,X) = \psi_l(d)\phi(X)\nu$, we can identify the structural hazards from the weighted unemployment distribution. In this case, the weights must be chosen to ensure a comparable distribution of observable characteristics across notice length, conditional on the censoring duration being greater than $\bar{D}$.

Appendix E Robustness

In this subsection, I present a series of robustness checks. Figure E.1 displays the data and estimated structural hazard for a sample that excludes individuals with less than 1 month of notice, while Figure E.2 illustrates the same for the unweighted sample. In both cases, estimates are quantitatively the same as the baseline estimates.

As shown in the paper, the specified Mixed Hazard model is non-parametrically identified. Figure E.3 presents the non-parametric estimate for the structural hazard, along with the baseline estimate that assumes a log-logistic functional form for the hazard. The non-parametric hazard declines even after benefit extension. However, similar to the baseline estimate, it increases going up to benefit exhaustion and does not fall below the initial hazard, contrary to the observed hazard. In the literature, it is common

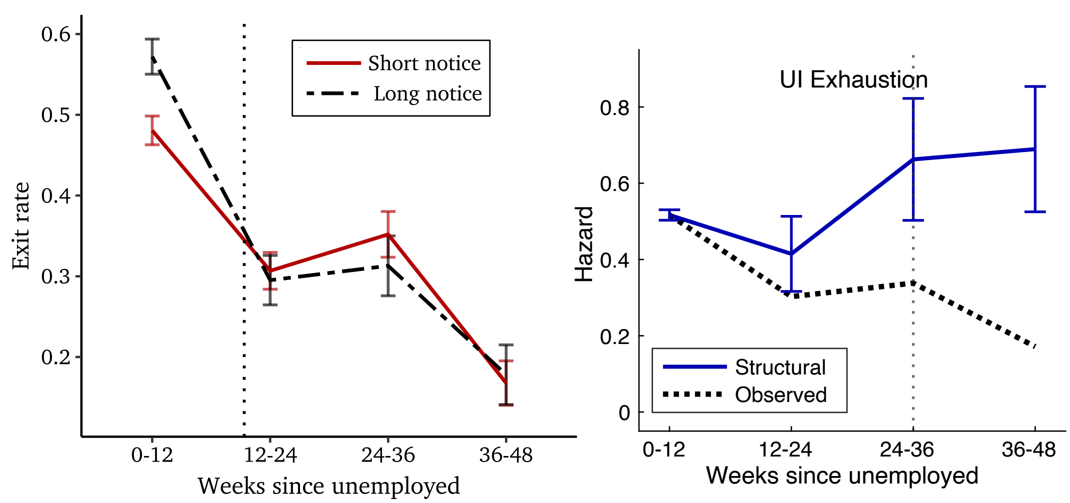
FIGURE E.1: DATA AND ESTIMATES WITH ALTERNATIVE NOTICE CATEGORIES



(A) EXIT RATE (B) ESTIMATED STRUCTURAL HAZARD

Note: Short notice refers to a notice of 1-2 months and long notice refers to a notice of greater than 2 months. Panel A presents the exit rate from the data separately for long and short-notice workers. The solid line in panel B shows the estimated structural hazard from the Mixed Hazard model, while the dotted line represents the average exit rate for both short and long-notice workers in the data.

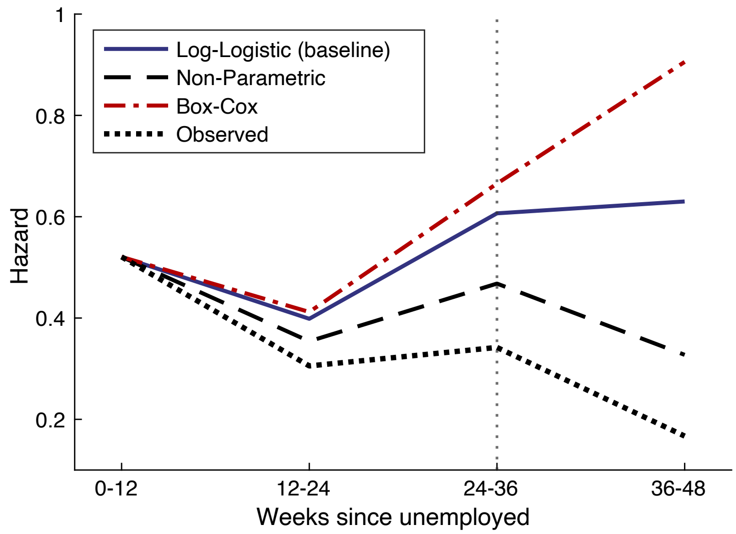
FIGURE E.2: DATA AND ESTIMATES USING THE UNWEIGHTED SAMPLE



(A) EXIT RATE (B) ESTIMATED STRUCTURAL HAZARD

Note: The figure presents data and estimates for the unweighted analytical sample. Panel A presents the exit rate from the data separately for long and short-notice workers. The solid line in panel B shows the estimated structural hazard from the Mixed Hazard model, while the dotted line represents the average exit rate for both short and long-notice workers in the data.

Figure E.3: Estimates with Different Functional Forms

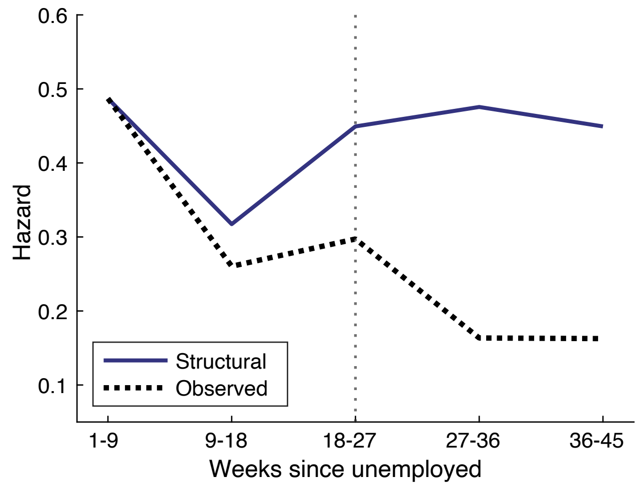


Note: Figure presents estimates for the structural hazard from the Mixed Hazard model under alternative parametric assumptions. The dotted line presents the observed exit rate from the data.

to impose a Weibull or a Gompertz hazard. However, I choose the log-logistic form because it allows the hazard to be non-monotonic. In Figure E.3, I also present estimates assuming the Box-Cox functional form, given by $\psi(d) = \exp\left[\frac{\alpha d^{\beta}-1}{\beta}\right]$. With $\beta \to 0$, this converges to the Weibull hazard, with $\beta = 1$ it is equal to Gompertz, and $\beta = 0$ implies a constant hazard. The estimates from this specification result in an increasing hazard.

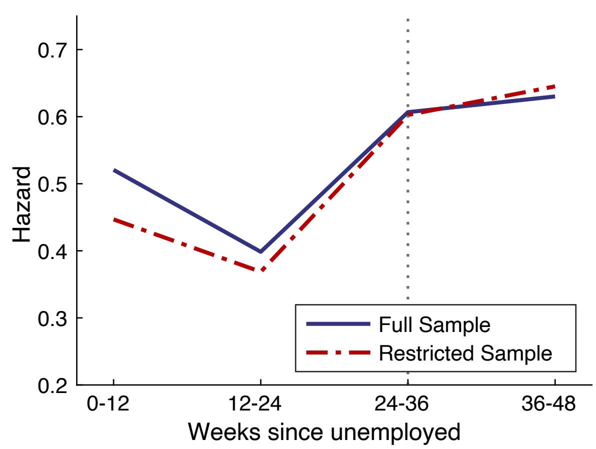
Figure E.4 presents estimates from the data with unemployment duration binned in 9-week intervals. The estimated structural hazard exceeds the observed hazard and follows a similar pattern to the baseline estimates. It rises more than the observed hazard until unemployment insurance (UI) benefits are exhausted and is constant after. Finally, Figure E.5 estimates the model separately for individuals who were displaced during years when UI benefits were potentially extended beyond 26 weeks. Two extensions during the sample period, first, from March 2002 to early 2004 through the Extended Unemployment Compensation (TEUC) legislation, and second, from July 2008 to the end of 2013 through the Emergency Unemployment Compensation (EUC) program.

Figure E.4: Estimates with Unemployment Duration Binned in 9-Week Intervals



Note: The figure presents estimates from the Mixed Hazard model using data with unemployment duration binned in 9-week intervals. The solid line presents the estimates for the structural hazard, while the dotted line presents the observed exit rate from the data.

Figure E.5: Estimates for Years with Extended Benefits



Note: The figure presents estimates from the Mixed Hazard model for a restricted sample of individuals who lost their jobs during times when unemployment benefits were possibly extended beyond 26 weeks. The restricted sample includes individuals displaced between 2001-2004 and 2008-2013. The estimated structural hazard for the full sample is also presented for comparison.

APPENDIX F GENERALIZATION

The main identification result in the paper relies on two crucial assumptions: (i) the notice length is independent of the worker type (conditional on observables), and (ii) the structural hazard after the initial period is identical regardless of notice length. In this section, I generalize the identification result and show that it is possible to identify structural duration dependence and the moments of heterogeneity distribution as long as we know how the structural hazard after the initial period, as well as the distribution of heterogeneity, varies across workers with different notice lengths. In particular, consider two lengths of notice and define κ_d as the difference between the d^{th} moment of ν conditional on l' and l as follows

$$\kappa_d = \mu_{l',d} - \mu_{l,d}$$

where $\mu_{l,d} = E(\nu^d|l)$. So κ_1 is the difference between the average type of workers with l' and l notice lengths. Additionally, define γ_d as the ratio of structural hazards at duration d for two lengths of notice,

$$\gamma_d = \frac{\psi_{l'}(d)}{\psi_l(d)}$$

Now if for some $\bar{D}$ we know κ_d for $d = 1, ..., \bar{D}$ and γ_d for $d = 2, ..., \bar{D}$, we can identify the first $\bar{D}$ structural hazards and moments of type distribution for each notice length up to scale.[4] To see why this is the case, note that for notice length l, the observed

[4]Alternatively, we could know κ_d for $d = 2, ..., \bar{D}$ and γ_d for $d = 1, ..., \bar{D}$. Also, in theory, the choice of defining γ_d and κ_d as a ratio or a difference does not impact the proof of identification. In this case, I define κ_d as a difference and γ_d as a ratio for the convenience of varying these parameters when examining the changes in estimates.

hazards at $d=1$ and $d=2$ can be written as:

$$\tilde{h}(1|l) = \psi_l(1)\mu_{l,1}$$
$$\tilde{h}(2|l) = \psi_l(2)\mu_{l,1}\left(\frac{1-\tilde{h}(1|l)(\mu_{2,l}/\mu_{1,l}^2)}{1-\tilde{h}(1|l)}\right)$$

As before, if we knew the extent of heterogeneity across workers, i.e. the variance of ν amongst l notice individuals, we would be able to infer structural duration dependence $\psi_l(2)/\psi_l(1)$ from observed duration dependence $\tilde{h}(2|l)/\tilde{h}(1|l)$. Now, we also observe the hazard conditional on notice length l', which is given by

$$\tilde{h}(2|l') = \gamma_2\psi_l(2)(\mu_{l,1}+\kappa_1)\left(\frac{1-\tilde{h}(1|l')\big((\mu_{2,l}+\kappa_2)/(\mu_{1,l}+\kappa_1)^2\big)}{1-\tilde{h}(1|l')}\right)$$

So now if we compare $\tilde{h}(2|l')$ to $\tilde{h}(2|l)$, as before, the difference between the two depends on $\mu_{2,l}$, however, now it also depends on γ_2, κ_1, and κ_2. So if we know γ_2, κ_1, and κ_2, we can still back out $\mu_{2,l}$. The intuition for the result is that we know how the structural hazards for different notice lengths at $d=2$ should vary if there was no heterogeneity. Then if we observe the structural hazards being different over and above what we would expect with no heterogeneity, we can attribute that to the presence of heterogeneity.

Theorem F.1. For some l, l', define $\kappa_d = \mu_{l',d} - \mu_{l,d}$ and $\gamma_d = \psi_{l'}(d)/\psi_l(d)$. Then for some $\bar{D}$, if $\{\kappa_d\}_{d=1}^{\bar{D}}$ and $\{\gamma_d\}_{d=2}^{\bar{D}}$ are known, then the baseline hazards $\{\psi_l(d), \psi_{l'}(d)\}_{d=1}^{\bar{D}}$ and the conditional moments of the type distribution $\{\mu_{l,d}, \mu_{l',d}\}_{d=1}^{\bar{D}}$ are identified up to a scale from $\{G(d|l), G(d|l')\}_{d=1}^{\bar{D}}$.

Proof. First note that we can write,

$$g(d|l) = \psi_l(d)\sum_{k=1}^{d} c_k(\boldsymbol{\psi_{l,d-1}})\mu_{l,k} \tag{2}$$

where $\boldsymbol{\psi}_{\boldsymbol{l,d-1}} = \{\psi_l(s)\}_{s=1}^{d-1}$, $c_k(\boldsymbol{\psi}_{\boldsymbol{l,0}}) = 1$, and

$$c_k(\boldsymbol{\psi}_{\boldsymbol{l,d-1}}) = \begin{cases} c_k(\boldsymbol{\psi}_{\boldsymbol{l,d-2}}) & \text{for } k = 1 \\ c_k(\boldsymbol{\psi}_{\boldsymbol{l,d-2}}) - \psi_l(d-1)c_{k-1}(\boldsymbol{\psi}_{\boldsymbol{l,d-2}}) & \text{for } 1 < k \le d \\ 0 & \text{for } k > d \end{cases}$$

Now we can prove the statement of the theorem by induction. First, note that the statement is true for $\bar{D} = 1$. To see this, note that

$$g(1|l) = \psi_l(1)\mu_{l,1} \qquad g(1|l') = \psi_{l'}(1)(\mu_{l,1} + \kappa_1)$$

We will normalize $\mu_{l,1} = 1$. Then we can solve for $\psi_l(1) = g(1|l)$ and $\psi_{l'}(1) = \frac{g(1|l')}{1+\kappa_1}$.

Now let us assume that the statement is true for $\bar{D} = d - 1$. Then we can identify $\{\psi_l(s), \psi_{l'}(s)\}_{s=1}^{d-1}$ and $\{\mu_{l,s}, \mu_{l',s}\}_{s=1}^{d-1}$ from $\{G(s|l), G(s|l')\}_{s=1}^{d-1}$. To complete the proof, we need to prove that the statement is true for $\bar{D} = d$ as well.

Denote $\Gamma_d = \prod_{s=1}^{d} \gamma_s$ and $\Psi_l(d) = \prod_{s=1}^{d} \psi_l(s)$. Now note that,

$$\begin{aligned} g(d|l) &= \psi_l(d) \sum_{k=1}^{d} c_k(\boldsymbol{\psi}_{\boldsymbol{l,d-1}})\mu_{l,k} \\ &= \psi_l(d) \left[\sum_{k=1}^{d-1} c_k(\boldsymbol{\psi}_{\boldsymbol{l,d-1}})\mu_{l,k} + c_d(\boldsymbol{\psi}_{\boldsymbol{l,d-1}})\mu_{l,d} \right] \\ &= \psi_l(d) \left[\sum_{k=1}^{d-1} c_k(\boldsymbol{\psi}_{\boldsymbol{l,d-1}})\mu_{l,k} + (-1)^{d-1}\Psi_l(d-1)\mu_{l,d} \right] \end{aligned}$$

From the above equation we can solve for $\mu_{l,d}$ as follows:

$$\mu_{l,d} = \frac{(-1)^d}{\Psi_l(d-1)} \left[\sum_{k=1}^{d-1} c_k(\boldsymbol{\psi}_{\boldsymbol{l,d-1}})\mu_{l,k} - \frac{g(d|l)}{\psi_l(d)} \right] \tag{3}$$

Using the fact that $\mu_{l',d} = \kappa_d + \mu_{l,d}$, we can write $g(d|l')$ as follows:

$$g(d|l') = \psi_{l'}(d)\left[\sum_{k=1}^{d-1} c_k(\boldsymbol{\psi_{l',d-1}})\mu_{l',k} + (-1)^{d-1}\Psi_{l'}(d-1)(\kappa_d + \mu_{l,d})\right]$$

By plugging in $\mu_{l,d}$ from equation (3) in the above expression, we can solve for $\psi_{l'}(d)$ as follows:

$$\psi_{l'}(d) = \frac{g(d|l') - \Gamma_d g(d|l)}{\sum_{k=1}^{d-1} c_k(\boldsymbol{\psi_{l',d-1}})\mu_{l',k} - \Gamma_{d-1}\sum_{k=1}^{d-1} c_k(\boldsymbol{\psi_{l,d-1}})\mu_{l,k} + (-1)^{d-1}\kappa_d\Psi_{l'}(d-1)}$$

Plugging this back in expression for $\mu_{l',d}$, we can solve for

$$\mu_{l',d} = \frac{(-1)^d}{\Psi_{l'}(d-1)}\left[\frac{g(d|l')\Gamma_{d-1}\sum_{k=1}^{d-1} c_k(\boldsymbol{\psi_{l,d-1}})\mu_{l,k} - \Gamma_d g(d|l)\sum_{k=1}^{d-1} c_k(\boldsymbol{\psi_{l',d-1}})\mu_{l',k} - (-1)^{d-1} g(d|l')\kappa_d\Psi_{l'}(d-1)}{g(d|l') - \Gamma_d g(d|l)}\right]$$

So as long as the denominators in the expressions for $\psi_{l'}(d)$ and $\mu_{l',d}$ are not zero we would have identification. □

We can see that with $\kappa_d = 0$ for $d = 1,..,\bar{D}$ and $\gamma_d = 1$ for $d = 2,..,\bar{D}$, the above theorem is equivalent to the result in the main text. Also, note that the theorem can more generally be applied to situations with other observable characteristics. For instance, with $\kappa_d = 0$ for $d = 1,..,\bar{D}$ and $\gamma_d = \gamma$ for $d = 1,..,\bar{D}$, the above is equivalent to the discrete MPH model. In the following subsection, I investigate how the estimates of structural hazard vary under different assumptions on κ_d and γ_d.

F.1 Implementation

In our estimation, we utilized two lengths of notice, <2 months (S) and >2 months (L). Let's define $\kappa_d = \mu_{L,d} - \mu_{S,d}$ and $\gamma_d = \psi_L(d)/\psi_S(d)$. For our baseline estimates, we assumed that the distribution of heterogeneity for individuals with these different notice lengths was identical, i.e., $\kappa_d = 0$ for all d. We also assumed that after the first period, the structural hazards for both the groups were the same, so $\gamma_d = 1$ for $d > 1$.

I now study how our estimates change if the underlying distribution of heterogeneity and/or the structural hazards after the initial period are different for workers with different lengths of notice. In particular, I perform the following three exercises.

1. Allow average type to vary

I relax the assumption that notice length is independent of a worker's type and let the mean of the heterogeneity distribution vary across the two groups. I assume that apart from the mean, the rest of the shape of the distribution for the two groups is identical. Since we have $\bar{D} = 4$, this implies that the 2nd, 3rd, and 4th central moment, the variance, skewness, and kurtosis, for the two groups are identical. The non-central moments would be impacted by scale changes, so all four κ_ds will be non-zero. Denote central moments by $\tilde{\mu}$. Note that, $\tilde{\mu}_2 = \mu_2 - \mu_1^2$. Then since we need $\tilde{\mu}_{S,2} = \tilde{\mu}_{L,2}$,

$$\mu_{S,2} - \mu_{S,1}^2 = \mu_{S,2} + \kappa_2 - (\mu_{S,1} + \kappa_1)^2 \to \kappa_2 = \kappa_1(\kappa_1 + 2\mu_{S,1})$$

Similarly, noting that $\tilde{\mu}_3 = \mu_3 - 3\mu_1\mu_2 + 2\mu_1^3$ and setting $\tilde{\mu}_{S,3} = \tilde{\mu}_{L,3}$, implies $\kappa_3 = \kappa_1(\kappa_1^2 + 3\kappa_1\mu_{S,1} + 3\mu_{S,2})$. And since, $\tilde{\mu}_4 = \mu_4 - 4\mu_3\mu_1 + 6\mu_2\mu_1^2 - 3\mu_1^4$, then setting $\tilde{\mu}_{S,4} = \tilde{\mu}_{L,4}$, we would have $\kappa_4 = \kappa_1(\kappa_1^3 + 4\kappa_1^2\mu_{S,1} + 6\kappa_1\mu_{S,2} + 4\mu_{S,3})$.

Now assuming $\gamma_d = 1$ for $d > 1$ and normalizing $\mu_{S,1} = 1$, I reestimate the model for 25 equidistant values for κ_1 in the interval $[-0.1, 0.1]$.[5] κ_2, κ_3 and κ_4 are defined as above. Residuals from this exercise are presented in panel A of Figure F.1. In panel B, I present the estimates for structural duration dependence for the value of κ that minimizes the residuals. The minimizing value of κ is close to zero, leading to an identical estimate for the structural hazard as the baseline.

2. Allow structural hazards after the first period to vary

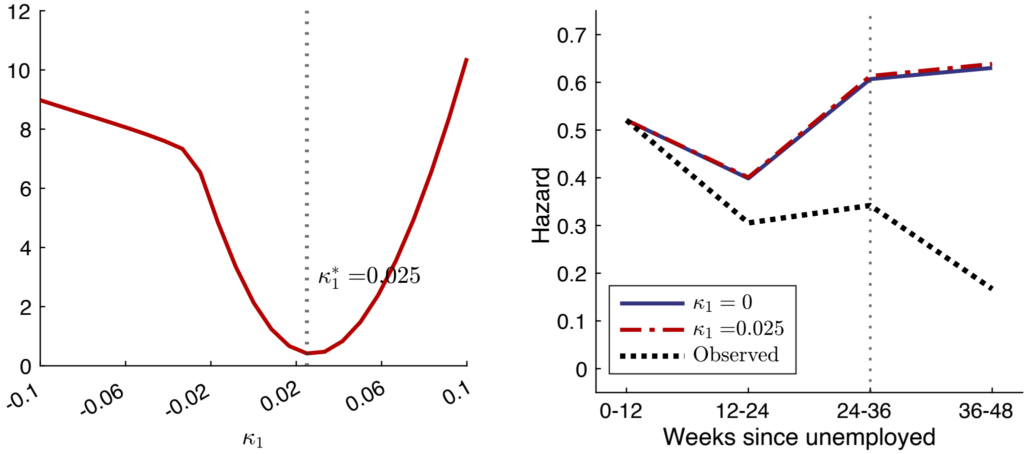
[5] For values beyond this interval, the model fit deteriorates drastically, and the estimated moments of the heterogeneity distribution blow up in either direction.

Now as in the baseline estimation, I assume notice length to be independent of worker type. But now we will allow structural hazards beyond the initial period to vary for workers with different lengths of notice up to some constant γ. This corresponds to assuming $\kappa_d = 0$ for $d = 1, .., \bar{D}$ and $\gamma_d = \gamma$ for $d = 2, .., \bar{D}$. I estimate the model for 25 equidistant values for γ in the interval $[0.95, 1.2]$. Results from this exercise are presented in Figure F.2. The results point towards the structural hazard being slightly greater for individuals with a longer notice, even beyond the first 12 weeks. As we can see from panel B of Figure F.2, this suggests that the baseline estimates might be underestimating the role of dynamic selection. The reason for this is that in the case that the structural hazard for long-notice workers is higher even beyond the initial period, the gap between the long and short-notice average exit rates due to composition would be greater than what we assumed in the baseline estimation.

3. Allow the average type and structural hazards after the first period to vary

Finally, I create a 20×20 grid for values of $\kappa \in [-0.1, 0.1]$ and $\gamma \in [0.95, 1.20]$. I reestimate the model for each point in the grid. Panel A of Figure F.3 presents the residuals for different values in the grid. While panel B of Figure F.3 presents estimates at the minimizing values. The results from the exercise point towards no mean differences between short and long-notice groups, but a higher structural hazard for long-notice workers beyond the initial period. This results in an estimate for the structural hazard that increases more than the baseline estimate.

FIGURE F.1: ALLOW AVERAGE TYPE TO VARY

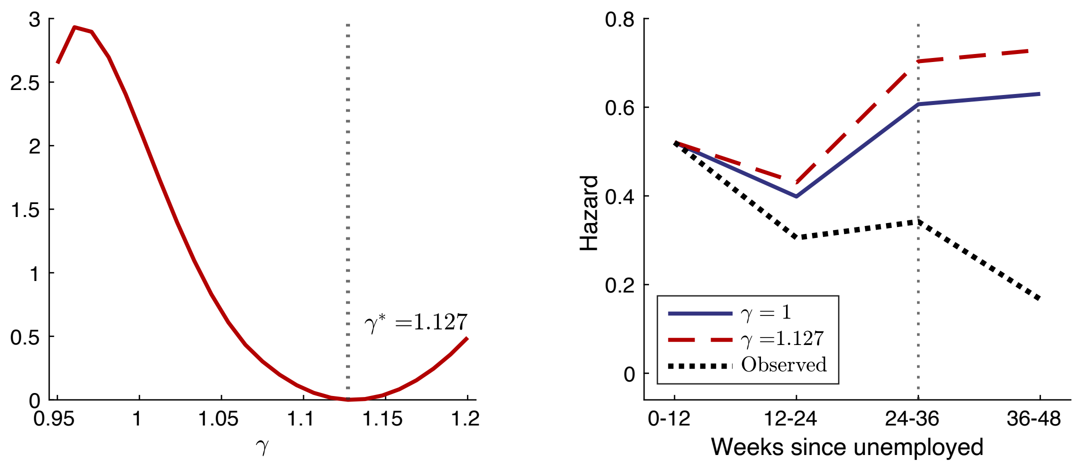


(A) RESIDUALS

(B) STRUCTURAL HAZARD

Note: The figure presents results from the estimation of a more generalized Mixed Hazard model, where the mean of the heterogeneity distribution for individuals with different lengths of notice is allowed to vary according to the parameter κ_1. Panel A presents the residuals from GMM estimation for different values of κ_1. Panel B presents the estimates of structural hazard for different values of κ_1.

FIGURE F.2: ALLOW STRUCTURAL HAZARDS AFTER THE FIRST PERIOD TO VARY

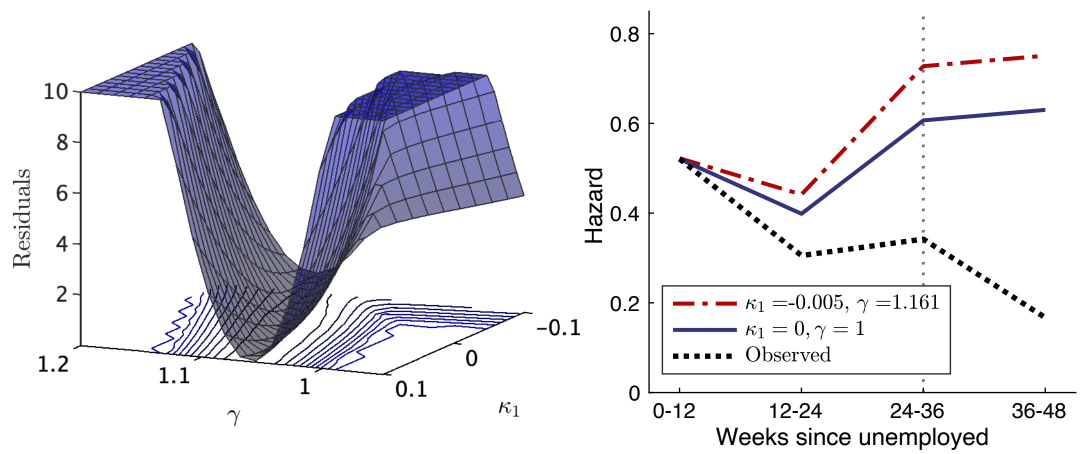


(A) RESIDUALS

(B) STRUCTURAL HAZARD

Note: The figure presents results from the estimation of a more generalized Mixed Hazard model, where the structural hazard after the initial period for individuals with different lengths of notice is allowed to vary according to the parameter γ. Panel A presents the residuals from GMM estimation for different values of γ. Panel B presents the estimates of structural hazard for different values of γ.

Figure F.3: Alternative Assumptions on Structural Hazards and Heterogeneity Distribution



(A) Residuals

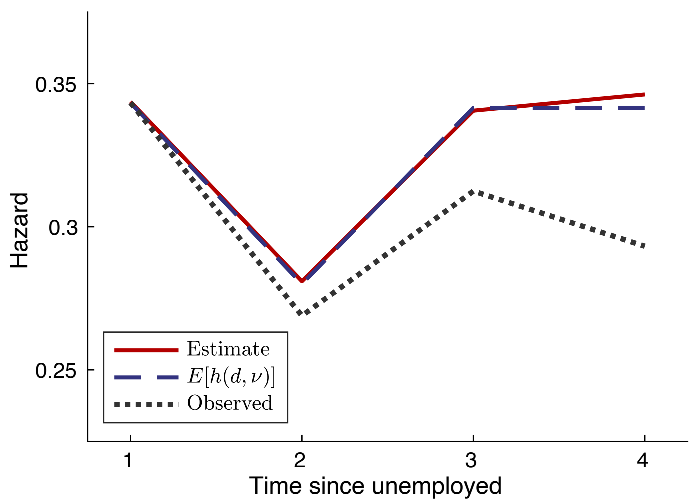
(B) Structural Hazard

Note: The figure presents results from the estimation of a more generalized Mixed Hazard model. The mean of the heterogeneity distribution for individuals with different lengths of notice is allowed to vary according to the parameter κ_1. The structural hazard after the initial period for individuals with different lengths of notice is allowed to vary according to the parameter γ. Panel A presents the residuals from GMM estimation for different values of κ_1 and γ. Panel B presents the estimates of structural hazard for the case where $\kappa_1 = 0$ and $\gamma = 1$ (solid line) and for the case when κ_1 and γ take values that minimize the residual in Panel A (dashed line).

Appendix G Search Model Simulation

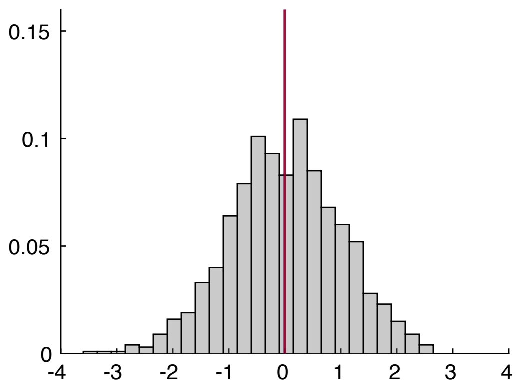
In this section, I simulate data from the search model presented in the main text. To incorporate multiple notice periods, I let the offer rate in the first period be different for long (L) and short (S) notice individuals. I set $\nu_H = 1$, $\nu_L = 0.5$ and $\pi = 0.5$, $\delta_L(1) = 1.25, \delta_S(1) = 1$, and $\delta(d) = 0.95$ for $d = 2, 3, 4$. The rest of the parameters are set as in the calibration of the model in the main text. I assume there are 2500 individuals, half of whom receive the L length notice. I simulate data on exit rates for this model 1000 times. The average of estimates for the structural hazard is presented in Figure G.1, while the distribution of the estimates is presented in Figure G.2.

Figure G.1: Simulation: Average Estimate

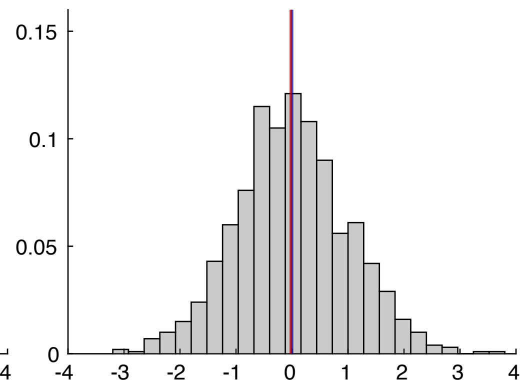


Note: The solid line presents the average estimate from 1000 simulations of the search model. The dashed line presents the structural duration dependence $\mathbb{E}[h(d|\nu)]$ implied by the model. While the dotted line presents the observed structural duration dependence $\mathbb{E}[h(d|\nu)|D \geq d]$ implied by the model.

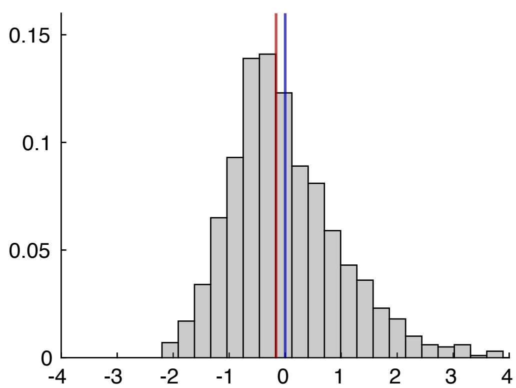
Figure G.2: Estimates using Simulated Data from the Search Model



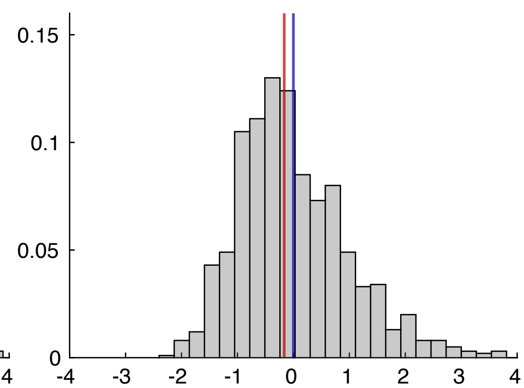
(A) $\psi(1)$



(B) $\psi(2)$



(C) $\psi(3)$



(D) $\psi(4)$

Note: The figure presents the distribution of estimates of structural duration dependence on simulated data from the search model. The vertical lines represent the mean and median of the distribution for each structural hazard.